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THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED BY

EDWARD L. NICHOLS

ERNEST MERRITT, and FREDERICK BEDELL

LXIII

MAY 1901

TABLE OF CONTENTS

Frontispiece

Surface Tension at the Interface of Two Liquids Determined Experimentally by the Method of Ripple Waves — FLOYD R. WATSON 257

An Improved Method of Rating Tuning Forks — JOHN O. REED 279

George Francis Fitzgerald — J. LARMOR 292

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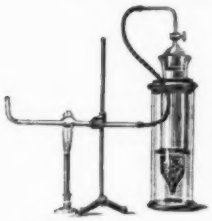

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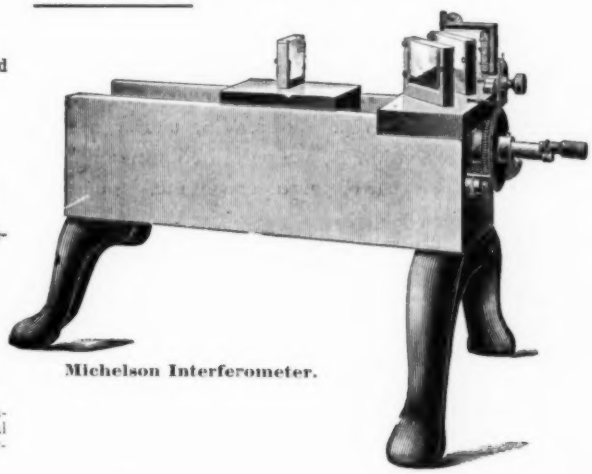

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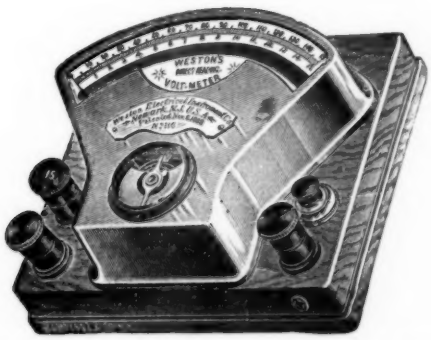

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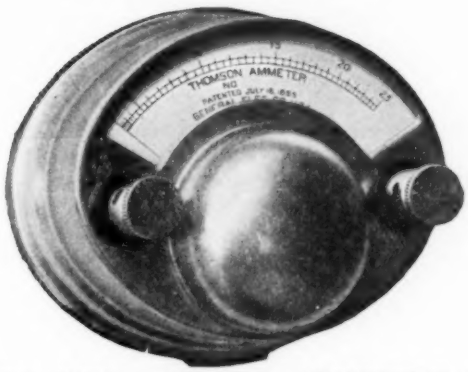

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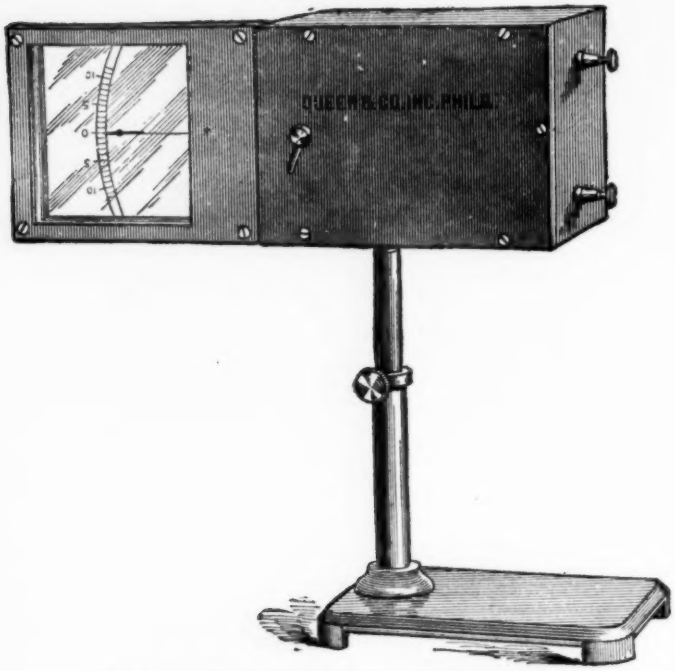

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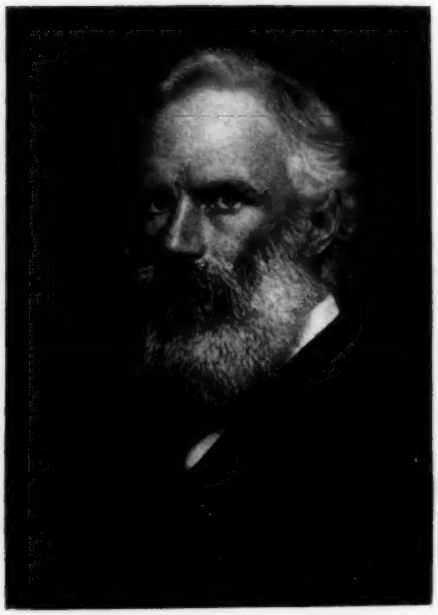

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Volume XII. *May, 1901.* *Number 5.*

THE

PHYSICAL REVIEW.

SURFACE TENSION AT THE INTERFACE OF TWO LIQUIDS DETERMINED EXPERIMENTALLY BY THE METHOD OF RIPPLE WAVES.

By Floyd R. Watson.

THE method of ripple waves used in determining surface tensions of liquids is well known. Thomson[1] advanced the theory that, when the length of waves on the surface of a liquid is small, the vibration of the waves depends chiefly upon capillarity. The formula deduced in accordance with this theory is as follows,

$$T = \frac{n^2\lambda^3}{2\pi}(\rho_1 + \rho_2) - \frac{g\lambda^2}{4\pi^2}(\rho_1 - \rho_2)$$

where T is the surface tension, λ the wave-length, n the frequency of the waves, g the acceleration of gravity, ρ_1 the density of the lower liquid and ρ_2 the density of the upper. In case the upper liquid be air, ρ_2 may be neglected, and the formula becomes

$$T = \rho_1\left(\frac{n^2\lambda^3}{2\pi} - \frac{g\lambda^2}{4\pi^2}\right).$$

Lord Rayleigh[2] was the first to use this theory to determine surface tension, his general method being to excite the surface of water periodically and view, by means of interrupted light, the waves formed; the flashes of light being isoperiodic with the waves —the so-called stroboscopic method. Later Dorsey,[3] Smith,[4] Bar-

[1] Phil. Mag., (4), 42, 375.
[2] Phil. Mag., (5), 30, 386.
[3] Phys. Review, 5, 170, 213.
[4] Proc. Roy. Soc. Edinb., 17, 115.

nett [1] and others used Rayleigh's method with more perfected apparatus.

During two years, 1898–1900, while at the University of California, the author devoted some time to the study of surface tension. The first part of the work was a verification of Lord Rayleigh's experiment, his method being used and his apparatus duplicated as far as possible. Later both the method and the apparatus were improved until consistent results were obtained. Further work on the same problem has been done by the author at Cornell University, where still better methods were used with more refined apparatus.

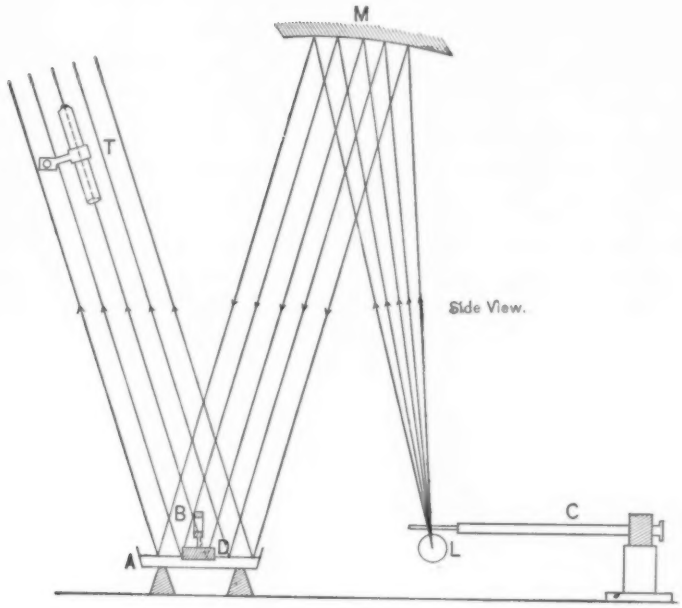
By means of measurements taken, surface tensions were calculated for mercury, water, petroleum, benzine and alcohol when these liquids were in contact with air. By a modification in the apparatus, measurements were taken giving interfacial tensions for mercury-water, mercury-alcohol, water-petroleum and water-benzole. The method was perfected for measuring surface tension between any two liquids that do not mix or unite chemically.

Description of Apparatus.

The apparatus and method used at the University of California will be described first and a table of results given. Later the work at Cornell will be described and comparisons made.

Figure 1 gives a diagrammatic sketch of the apparatus used in generating the waves and measuring their length. The waves were generated on the liquid in the pan A by means of the glass microscope slide D attached to the end of the tuning fork prong B. Fork B is attached to fork C by a wire E, the ends of the wire being fastened securely to the prongs of the forks as shown in the figure. Fork C is run electro-magnetically, the amplitude of vibration of its ends being about $\frac{3}{4}$ cm. Fork B is vibrated by the mechanical impulses sent along the wire E, both forks thus vibrating in unison. At first fork B was run by a shunted circuit from fork C, but the vibrations of the forks were then not always in unison, nor were their movements steady. An attempt was made

[1] Phys. Review, 6, 257.



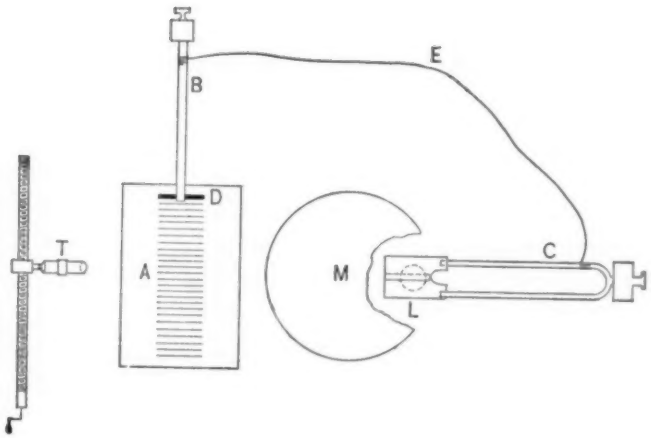


Fig. 1.

Sketch of apparatus.

to vibrate fork *B* without shunting the current from *C*. This was done by placing the fork *B* so that its prongs were in the same plane with those of *C* and connecting adjacent prongs with a taut string, fork *C* being run electro-magnetically as before. This method worked well even after the string became somewhat loose. The last fact suggested to the author the plan of connecting the forks with a wire, so that they could be placed more conveniently for the purposes of the experiment. The plan was found to be successful—the forks vibrating steadily and in unison when separated by a distance so great as 6 feet. Later at Cornell, the author replaced fork *B* by a stick of wood, which vibrated equally as well as the fork.

The light was given by a 50 candle-power incandescent lamp whose filament was wound in the shape of a hollow cone, giving an intense concentrated source of illumination. The light was interrupted by the fork *C*, passed to the concave mirror *M*, and was rendered parallel by reflection, after which it was reflected from the surface of the liquid to the telescope *T*. The light was made intermittent by means of two small strips of wood attached to the prong ends of the fork *C*. The strips overlapped when the fork was at rest, and let the light through once each vibration when the fork prongs were in their position of extreme outward swing, thus making the flashes of light isoperiodic with the waves generated by fork *B*.

The telescope was moved by an endless screw so that its successive positions were parallel. The concave mirror was of glass, its rear surface being the reflector. Its diameter was about 30 cm. and its focal length 84 cm. The most satisfactory pan for holding the liquid was one of galvanized iron 24 by 27 cm. and 3 cm. deep. To prevent the vibrations carried by the floor from disturbing the surface of the liquid, the pan was set on a small wooden frame which had rubber tubing nailed to its upper surface, and had each of its four supports resting in a box of sand.

When the apparatus was properly arranged for taking measurements, the source of light, the axes of the telescope, mirror and fork *C* should lie in a plane perpendicular to the liquid, and the waves on the surface should be parallel to this plane.

Interfacial Waves.

Waves at the interface of two liquids such as mercury and water were obtained by having the glass vibrator on the prong of fork *B* immersed just below the mercury-water surface. The waves on the upper surface of the water were found to distort the appearance of waves at the interface, so a thin strip of glass was placed close to the vibrator and parallel to it with its lower edge immersed in the water surface, thus stopping the surface waves and allowing a clear view of the interfacial waves.

In the case of two liquids, such as water and oil, it was found that the reflection of light from the upper surface was so strong as to drown out the view of the waves at the interface. To remedy this trouble a piece of plate glass was suspended nearly parallel to the interfacial surface with its lower side just submerged in the oil. This arrangement caused the light striking the upper surface of oil to be reflected at a different angle from that of the light reflected from the interface, hence the waves could be seen. (See Fig. 2.)

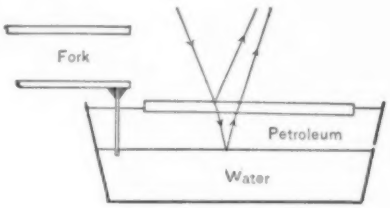


Fig. 2.

Method of Taking Measurements.

Everything that the liquid was to touch was thoroughly cleaned, first with acid, then with alcohol and finally with running tap water. The liquid whose tension was to be determined was poured in the pan to a depth of at least a centimeter, the glass vibrator on the end of the fork was adjusted so that it dipped just below the surface and was perpendicular to it. The vibration of the fork was regulated until the waves were steady. A narrow strip of German plate glass, ruled crosswise at certain known distances, was placed parallel to the surface of the liquid and as close as possible to the waves without touching them. By means of the traveling telescope the number of waves were then counted between the ruled cross lines. Having the ruled glass plate so close to the surface threw out any error due to the light rays not being parallel, or an ac-

cidental shifting of the telescope. Usually from 30 to 40 waves were counted and tenths of waves estimated. The readings were verified by running the telescope backwards and taking the readings again in the reverse order. Reflected waves were met with at times, but they were reflected from the end of the pan which was parallel to the glass vibrator, and hence caused no distortion.

To determine n, the frequency of the waves, it is only necessary to determine the period of either fork, the periods of the forks and the waves being the same. The method first used was a chronograph method, where the fork's vibrations were traced by the side of a standard time circuit trace. Trouble was encountered in this method, in that the shunt from the interrupting fork C caused the fork to run unsteadily and probably change its period slightly. To obviate this difficulty, an auxiliary circuit was established (see Fig. 3). A small light strip of hard rubber, AB, was screwed to the fork and at A a small wire was threaded through. Each end of this wire dipped into a small mercury cup. As the fork vibrated the wires dipped into the mercury once each vibration closing the auxiliary circuit $ADCEA$ which made the fork's trace on the chronograph. The strip of rubber was left on the fork so as not to alter the period.

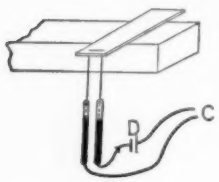


Fig. 3.

Another more convenient method used was to place a standard Koenig fork with a vibration microscope just over the interrupting fork and parallel to it, so that the flashes of light were viewed through the microscope when both forks were in motion. The light appeared to move back and forth, corresponding to beats. The number of these excursions, or beats, in a given time was counted, and knowing the pitch of the standard fork, the frequency of the interrupting fork, and hence that of the waves, was calculated.

The value taken for g was 980 cm. per sec. per sec., though the local value at Berkeley was 979.8, the error being small compared with the error in measuring λ. The density for water was taken to be 1, and for mercury 13.6. Temperatures and densities were determined by standard instruments. To reduce the value of the tensions to 0° C. the following formula[1] was used:

[1] Dorsey, Phil. Mag., Nov., 1898, page 388.

$T_t = T_0 (1 - at)$ where t is the observed temperature, T_0 the tension at 0° C. and $a = .0020$.

In the following table ρ gives the density, t the temperature in degrees Centigrade, λ the wave-length in centimeters, n the vibration frequency of the fork, and T_0 the surface tension in dynes per cm. at 0° C.

TABLE I.

Date.	Liquid.	ρ	t	λ	n	T_0
Aug. 10	Distilled water	.997	16.6	.5200	58.75	77.93
" 14	" "	"	16.8	.5263	"	76.24
" 15	" "	"	16.6	.5263	59.00	76.21
" 17	" "	"	18.7	.5263	"	76.54
" 17	" "	"	18.7	.5264	"	76.56
" 18	" "	"	16	.5263	"	76.11
" 24	" "	"	19.8	.3300	115.50	76.72
" 13	Mercury	13.6	18.2	.4039	58.75	433.25
" 15	"	"	16.6	.3967	59.00	432.01
" 17	"	"	16	.3994	"	440.18
" 17	"	"	18.7	.3984	"	439.22
" 18	"	"	16	.4000	"	442.25
" 20	"	"	18.1	.3984	"	438.67
" 22	"	"	18.7	.2500	115.5	447.93
" 10	Petroleum	.7875	16.6	.4076	58.75	26.93
" 13	"	"	18	.4098	"	27.38
" 14	"	"	16.8	.4032	59.00	26.28
" 17	"	"	16	.4016	"	25.92
" 17	"	"	18.7	.4016	"	26.06
" 21	"	"	18.7	.2538	115.25	27.14
" 15	Benzine	.753	15.7	.3981	59.00	24.26
" 15	"	"	16.6	.3927	"	22.41
" 18	"	"	16	.3891	"	22.45
" 15	Mercury-water		16.6	.3670	"	369.50
" 17	"		18.7	.3663	"	369.37
" 18	"		16	.3636	"	367.87
" 21	"		19	.2272	115.50	361.48
" 22	"		18.7	.2294	115.25	372.30
" 22	"		19.2	.2273	"	364.10
" 24	"		18.4	.2288	"	368.63
" 15	Mercury-alcohol		15.7	.3641	59.00	357.05
" 17	"		16	.3644	"	356.00
" 20	"		18.1	.3650	"	350.97

CHANGES IN APPARATUS AT CORNELL UNIVERSITY.

It was found necessary to suspend part of the apparatus in order to get rid of extraneous vibration. The pan holding the liquid, and

the fork generating the waves were placed on a slab of iron weighing about 60 pounds. This, in turn, rested on a heavy platform of wood which was suspended from the ceiling by four stout cotton cords. In place of the endless screw, a dividing engine was used to support the telescope, greater accuracy being obtained in measuring λ. A carefully-made silver-on-glass astronomical mirror of 8 inches diameter and 12 feet radius was substituted for the rough lecture mirror. The light was interrupted twice per vibration instead of once as before. This effect was obtained by using the wooden strips at the ends of the fork prongs as before, and cutting a slit in each strip parallel to the axis of the fork so that the light passed through when the fork was at rest, or in its equilibrium position when vibrating. The change which was most convenient was the substitution of a stick of wood for the fork which generated the waves. It was found that the stick, after being clamped to hold it steady, would take up the vibrations as readily as the fork; so the fork was discarded and the glass plate, which generated the waves, was fastened by a suitable device to the end of the stick. Several samples of wood of different kinds and dimensions were tried, the one finally selected being a strip of chestnut 12 inches long, $\frac{3}{4}$ inch wide, and about $\frac{1}{8}$ inch thick. It was clamped at a point near its middle.

METHOD OF CLEANING.

Cleanliness being an essential feature of the experiment, great care was taken in eliminating impurities. The glass and the pan were first cleaned with acid, an alcoholic solution of potash, and running tap water. They were then coated with a moist layer of "Bon Ami" (a commercial article for cleaning glassware) and dried thoroughly. This layer in drying took up any impurities that adhered to the glass, so that when the dishes were thoroughly rubbed with clean cloths the surface was clean. The pan and glasses were then rinsed with running tap water and finally with distilled water. Care was taken in all cases not to let the fingers touch the glass or pan where the liquid was to come in contact.

Mercury was cleaned by pouring it into a bottle containing concentrated sulphuric acid and a pasty layer of mercurous sulphate. The sulphuric acid, being the top layer, took out organic impurities,

and water; the next layer of mercurous sulphate acted on metallic impurities present such as compounds of lead, tin, zinc, etc. As these impurities floated on the surface of the mercury, the mercurous sulphate acted readily on them forming zinc-sulphate, lead-sulphate, etc., throwing down metallic mercury. The mercury used was drawn from the bottom of the bottle and found to be free from impurities.

To avoid contamination of the air, each liquid was cleaned just before a measurement was taken by sweeping its surface with a glass slide. This slide could be swept across the pan and held in its final position by means of the pressure against the sides of the pan of strips of rubber attached to the ends of the glass. In this way all the contaminations could be swept into one end and confined there, leaving a clean surface for the waves.

Changes in Method of Measurement.

The telescope was focused successively on three neighboring crests a wave-length apart and readings taken. It was then moved along and focused on the eleventh wave and readings taken for three crests as before. This gives three determinations of ten wave-lengths. At times the waves were not damped out so quickly and more than ten waves could be counted.

Many different samples of glass were tried for generating the waves. In order to get a steady pattern it is necessary that the junction of glass and liquid be a straight line, that is, the liquid should wet the glass. French plate glass was found to work well, but was too heavy for the stick vibrator, the vibrations obtained being too small. A thin piece of plane glass was found to give steady waves and that was used. In February the glass plate vibrator was discarded for a small pointed tube of glass which gave circular waves of a much more steady pattern. The lengths of waves were compared for the two cases, and were found to be the same when the point source was very small.

A new method was used to measure the frequency of the fork. A metal disc 12 inches in diameter and having four slits cut out as shown in Fig. 4 was screwed firmly to the shaft of a small electric motor. The disc was then placed horizontally over the vibrating

fork and rotated until the fork prongs appeared to stand still. A speed-indicator was attached to the motor shaft and the number of revolutions in a given time determined by means of a stop-watch. The author was assisted in this determination by Mr. Rands, who read the stop-watch. One of the difficulties was to keep the disc in unison with the fork. This was done by having a little more current than was necessary and slowing the speed of the disc by using the finger as a brake. With a little practice, the author succeeded in having the disc and fork run in unison for any desired time. Three successful measurements were made, the readings being as follows—

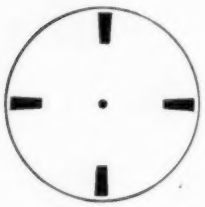


Fig. 4.

1. 3700 revolutions of disc in 5 minutes, 37 seconds.
2. 3600 " " " " 5 " 27.3 "
3. 2100 " " " " 3 " 11 "

giving respectively 10.979, 10.999, and 10.990 revolutions of the disc per second.

The stop-watch was then compared with the standard clock of the Physics Department and found to gain 43.5 seconds in 1 hour and 8 minutes. Appropriate corrections gave the average number of revolutions per second as 11.365, or the number of views through the slits as 4×11.365 apparently determining the frequency of the fork to be 45.46. Previous determinations by means of a standard fork had given the frequency as 88.5 approximately, hence the preceding value is one-half the real value. The conclusion [1] was drawn that the fork was seen every *second* vibration, and that its frequency was 8×11.365, or 90.92 vibrations per second. The accuracy of the working of the dividing engine was tested by Mr. Barnett [2] and also by the author and found to be correct. Densities and temperatures were determined as usual with standard instruments.

The following table gives the results obtained under the new conditions :

[1] Rayleigh, Phil. Mag., 16, 53.
[2] PHYS. REVIEW, 6, 257.

TABLE II.

Date.	Liquid.	ρ	t	λ	n	T_0
Nov. 23	Distilled water.	1	16.8	.3926	90.92	78.43
Dec. 1	" "	1	15.	.3920	"	77.77
" 1	" "	1	15.	.3923	"	77.95
Feb. 25	" "	1	11.2	.3927	"	77.55
" 25	" "	1	11.4	.3918	"	77.08
" 25	" "	1	11.4	.3912	"	76.72
" 25	" "	1	12.4	.3916	"	77.12
" 25	" "	1	12.2	.3923	"	77.50
Nov. 16	Tap water.	1	10.8	.3911	"	77.59
" 16	" "	1	11.2	.3915	"	76.87
" 17	" "	1	10.2	.3913	"	76.59
" 19	" "	1	8.2	.3916	"	76.27
Feb. 27	" "	1	6.0	.3914	"	75.82
" 27	" "	1	6.0	.3912	"	75.88
" 27	" "	1	6.0	.3916	"	76.12
Nov. 22	Mercury	13.6	17.5	.3051	"	494.07
" 22	"	"	"	.3042	"	489.55
" 22	"	"	"	.3044	"	490.60
" 22	"	"	"	.3033	"	485.20
" 23	"	"	19.8	.3033	"	487.51
" 23	"	"	"	.3047	"	494.39
" 23	"	"	"	.3038	"	490.07
" 23	"	"	"	.3022	"	482.07
" 23	"	"	"	.3037	"	490.19
" 26	"	"	"	.3026	"	485.10
Dec. 6	"	"	16.	.3002	"	468.65
" 7	"	"	17.5	.3023	"	480.41
" 8	"	"	16.	.3072	"	503.34
" 10	"	"	12.2	.3028	"	477.50
Nov. 23	Mercury-water		19.8	.2719	"	377.96
" 26	"		19.8	.2725	"	380.55
Dec. 6	"		16.	.2702	"	364.12
" 7	"		17.5	.2712	"	373.21
" 8	"		16.	.2706	"	365.77
" 10	"		12.2	.2728	"	374.32
Jan. 26	"		15.	.2730	"	378.88
" 26	"		15.	.2744	"	385.78
Feb. 4	"		11.5	.2720	"	371.96
" 4	"		11.5	.2732	"	376.71
" 12	"		15.5	.2710	"	370.84
" 15	"		14.	.2714	"	371.39
" 18	"		19.4	.2716	"	376.46
" 18	"		19.3	.2706	"	372.24
" 20	"		14.8	.2728	"	377.19
" 20	"		18.4	.2727	"	380.35
" 21	"		16.4	.2731	"	380.57
" 22	"		12.8	.2722	"	373.48
" 19	Mercury-alcohol	.825[1]	13.5	.2697	"	364.27
" 19	"	"	15.	.2697	"	365.02
" 20	"	"	17.5	.2722	"	378.36
" 26	"	"	11.	.2715	"	369.51
" 27	"	"	11.3	.2705	"	366.00

[1] Density of alcohol.

COMPARISON OF RESULTS.

It is first to be noted by the formula

$$T = \rho\left(\frac{n^2\lambda^3}{2\pi} - \frac{g\lambda^2}{4\pi^2}\right)$$

that an error in n is multiplied by two and an error in λ by three approximately in calculating T. The accuracy of the work is best judged then by an inspection of the values of λ, for the error in measuring n is very small. The Cornell measurements (Table II.) are selected for inspection as being more reliable. The measurements of λ on distilled water give an extreme error of less than $\frac{1}{3}$ of 1% and a variation from the mean of a little more than $\frac{1}{5}$ of 1%. The measurements for mercury are naturally not so good, giving an extreme error of $2\frac{1}{3}$% and a variation from the mean of $1\frac{1}{5}$%. Mercury-water measurements are more constant, giving an extreme variation of $1\frac{1}{2}$% and a variation from the mean of $\frac{2}{3}$ of 1%. The percentage of error is somewhat smaller than these figures indicate, in that wave-lengths for different temperatures are compared.

Comparing the results of Table II. with those of Table I. it is seen that the values for distilled water tensions are nearly the same, the Cornell tensions being about $\frac{3}{4}$ of a dyne greater. The tension of tap water is nearly as high as that of distilled water. The method pursued in getting these tensions was to rinse the pan thoroughly with running tap water, then draw off enough for measurement, and determine the wave-length as soon as possible. This method of getting a clean liquid by drawing it from the body of the supply rather than by pouring from the surface is in accord with experiments of Lord Rayleigh[1] and also of Mr. T. P. Hall.[2] Cornell measurements of mercury tensions give higher results throughout, due, without doubt, to the precautions taken in cleaning the surface and to the method of eliminating impurities. The values for mercury-water tensions show less difference than mercury-air tensions. The best interpretation here seems to be that most of the contamination from the air is probably moisture, so that water placed in contact with two samples of mercury—one exposed to the air and the other not—would

[1] Phil. Mag., (5), 33, p. 363.
[2] Phil. Mag., (5), 36, pp. 411–412.

give nearly the same interfacial tensions. This interpretation is shown by the results of Table II. The mercury-water tensions taken in November were on mercury previously exposed to the air, while the measurements in February were on a mercury-water surface where the mercury was drawn off *under* water, and not exposed at all to the air. The measurements in the two cases give practically the same results. Comparison of mercury-alcohol tensions show Cornell measurements to give higher results. This is probably due, as before stated, to better precautions in matter of cleanliness. The alcohol used at Cornell was distilled from the best commercial alcohol obtainable. The mercury used at the University of California was distilled in an ordinary vacuum still and filtered through clean cheese-cloth to remove motes from its surface, while the mercury used at Cornell was cleaned with sulphuric acid and mercurous sulphate, and was drawn off fresh for each measurement.

Other Interfacial Tensions.

Attempts have been made to measure surface tensions between mercury-petroleum and mercury-benzine, but a contaminating film was formed in each instance, so that no measurements were carried out.

Measurements have been taken giving tensions between water-petroleum and water-benzole. The results first obtained showed great variation. Several difficulties prevented accurate measurements being taken, first, a strong damping of the waves so that only four or five waves could be counted; and second, an irregularity in the wave pattern causing a variation in the measurement of the wave-length. (See Fig. 10.) It has already been stated that to get a steady pattern of the waves the line of junction of glass and liquid must be a straight line. This condition was not obtained with a water-petroleum surface when a plane glass vibrator was used—the result each time being a "crinkly" line. Later, however, a steady wave pattern was obtained by using a sharp-pointed glass tube as the source of disturbance. A third difficulty was experienced because of an insufficient illumination of the interfacial waves, the index of refraction of water being so nearly that of oil that but little light was reflected from their plane of separation.

To remedy this trouble an incandescent lamp of 100 candle-power was substituted for the one having 16 candle-power, and a mirror of shorter focus was substituted for the astronomical mirror. Changing the mirrors introduced a new source of error in that the new mirror, being roughly made, did not reflect the light in parallel rays. However, it was necessary to have more illumination in order to see the waves distinctly enough for measurement, so the change was made.

The usual precautions in regard to cleanliness were observed, the glasses and porcelain dish being cleaned with a concentrated nitric acid and a strong alcoholic solution of potash, besides being thoroughly rinsed with running tap-water and finally with distilled water. The benzole, petroleum and turpentine used were the best commercial article, and no attempt was made to purify them. Each liquid was placed in a thoroughly clean separatory funnel, and was drawn off from the bottom to prevent surface contamination.

Measurements were taken under these conditions, the liquids being arranged as shown in Fig. 2. Results are given in Table III.

TABLE III.

Date.	Liquids.	ρ	t	λ	n	T_0
April 2	Water-benzole.	.753	17.9	.2460	90.92	35.15
" 2	" "	"	15.6	.2380	"	31.75
" 3	" "	"	15.5	.2400	"	32.47
Jan. 22	Water-petroleum.	.787	17	.2428	"	34.36
March 2	" "	"	17	.2445	"	35.39
April 5	" "	"	19	.2420	"	34.18
" 5	" "	"	14.8	.2465	"	36.11
" 9	" "	"	17	.2480	"	36.79
" 9	" "	"	17	.2450	"	35.46

The density, ρ, here indicates the density of the lighter liquid, the density of water being taken as 1. The measurements of April 5 and 9 for water-petroleum were taken on petroleum that had been previously shaken with mercury to remove sulphur compounds. The results obtained show greater variation than do measurements for the tension between a single liquid and air. This is to be expected when the existing difficulties are considered. Each result recorded is the average of three or more measurements. Benzole-

water tensions were obtained to ascertain if a definite compound, such as benzole, would give a more constant tension with water than a mixture like petroleum. Each set of measurements has about the same range of variations so that by this method one tension is no more constant than the other.

Verification of Results.

These results were verified by two other methods. The first method involved the use of the Jolly balance. Surface tension was calculated by measuring the pull of a liquid film on the spring of the balance. The film was formed by means of a wire rectangle which was first immersed below the surface of the liquid and then drawn up by the spring. Two readings were taken, one when the rectangle was submerged and the second when the liquid film broke. The difference between these readings divided by twice the width of the rectangle and multiplied by the force in dynes necessary to stretch the spring one division of the scale gave the surface tension.

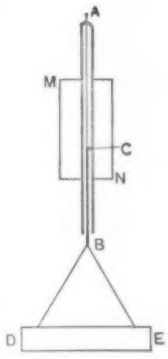


Fig. 5.

The second method was similar in principle. The pull of a suspended liquid film was measured by the sucking force of a coil of wire carrying a current.[1] (See Fig. 5.) *MN* represents the coil of wire which is wound about the small glass tube *AB*. *CB* is a bit of No. 26 soft iron wire, to which is soldered the piece of No. 33 copper wire, *CA* and the suspended rectangle *DE*. This core was hung inside the small glass tube, and prevented from falling by the small globule of solder at *A*, though a free upward movement was allowed.

Knipp found that the current necessary to raise the rectangle was proportional to the surface tension of the liquid used. The plan followed by the author was to immerse the rectangle in water and note the current necessary to raise the core. The rectangle was then raised until its upper edge was just below the surface of the liquid. The current was turned on, causing the rectangle to rise and form a film which broke after it had been stretched sufficiently.

[1] Knipp, Phys. Review, 11, 129, 1900.

The current strength was noted at this moment of breaking. The difference between the two readings taken was assumed to be proportional to the surface tension of water. The rectangle was again submerged just below the water surface and another liquid, such as petroleum, poured on the water; the current was turned on and its strength noted as the interfacial film broke. Finally the tension of the upper liquid was determined in the same way as that of water.

The glass beakers used to hold the liquids were cleaned as usual with alcohol, potash, nitric acid and water. The wire rectangle was of brass and was cleaned by the action of acid, all the outside layer being eaten off and the new surface thoroughly rinsed with water. It was found that a thin sheet of copper or platinum foil could be used in place of the brass wire rectangle, the results in each case being practically the same. Table IV. shows the results obtained.

TABLE IV.

Method used.	Water Tension.	Water-petroleum Tension.	Petroleum Tension.	Water Tension.	Water-benzole Tension.	Benzole Tension.
Jolly balance.	75.73	44.74	28.10	75.1	35.51	27.2
	73.99	43.89	28.90	74.9	36.20	30.6
	72.30	42.49	25.36			
Coil of wire.	75	41.4	27.2			
	75	38.3	29.5			
	75	33.8	28.7			

Interfacial tensions between water and benzole are the same as values obtained by ripples within errors of observation of the Jolly balance method. The seeming discrepancy in the interfacial tensions between water and petroleum puzzled the author for some time until a solution of the difficulty was found in an article by Mr. T. P. Hall,[1] in which he states, concerning the formation of liquid films, "Even when the bar is excessively thin a distinct maximum of weight is noticeable as the frame is raised, just before the true double film is formed. The bar must be raised from five to ten millimeters before a true film is formed, the tension not being correctly found at a less height."

[1] Phil. Mag. (5), 36, 385.

The height to which the rectangle was raised before the interfacial film broke was about 0.3 cm. as nearly as the author can remember from observation; therefore, according to Mr. Hall, the measurement should be high, which harmonizes with the results found. Mr. Hall shows that the tension can be found in terms of this maximum weight. The author has modified Mr. Hall's method to suit the present case and has found a verification of the measurements taken on ripple waves. The following paragraphs give details.

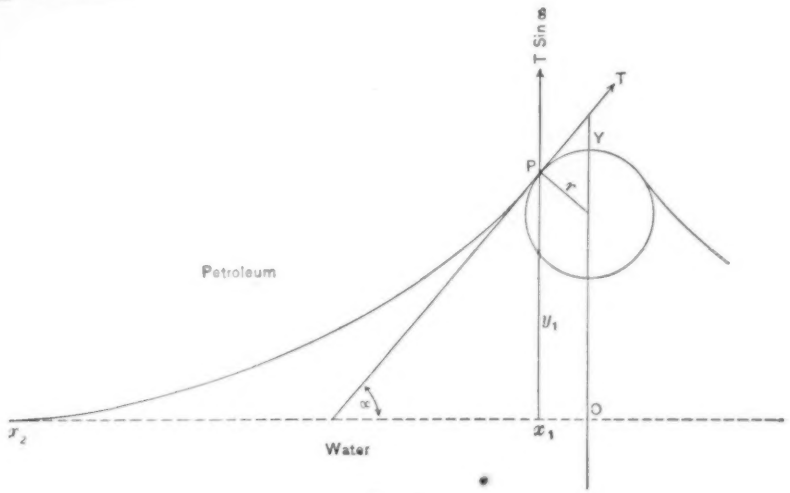


Fig. 6.

Fig. 6. is a vertical cross-section of the liquids, the circle representing the upper bar of the wire rectangle and Px_2 the line of separation of the two liquids. The weight of the mass above the interfacial level is desired. Taking a slice of unit thickness the weight of the mass from x_1 to x_2 is equal to the vertical component of the tension at the point P. The weight of the strip $OYPx_1$, may be taken with sufficient accuracy to be equal to the weight of the wire (immersed in water) plus $y(r \sin a)g \cdot (\rho_1 - \rho_2)$ where r is the radius of the wire, g the acceleration of gravity, ρ_1 the density of water and ρ_2 the density of petroleum. Thus the entire weight of the lifted mass

$$W = \omega + 2\,T \sin a + 2rgy(\rho_1 - \rho_2) \sin a$$

where ω is the weight of the brass wire when immersed in water.

In the experimental work this weight, w, was accounted for by subtracting the first reading from the second, so that there is left to calculate then only

$$W = [T + ryg(\rho_1 - \rho_2)]\ 2 \sin \alpha.$$

It yet remains to find an expreseion for sin α. It has already been stated that the vertical component of the tension supports the liquid, of unit thickness, included in the boundary $x_2\ P_1\ x_1$ thus

$$T \sin \alpha = g(\rho_1 - \rho_2) \int_{-x}^{x} y dx$$

from which

$$\frac{dx}{d\alpha} = \frac{T \cos \alpha}{gy(\rho_1 - \rho_2)} = \frac{c^2}{4y} \cos \alpha$$

where

$$c_2 = \frac{4T}{g(\rho_1 - \rho_2)}.$$

Further, as $\tan \alpha = \dfrac{dy}{dx}$

$$\frac{dy}{d\alpha} = \frac{dy}{dx} \cdot \frac{dx}{d\alpha} = \tan \alpha \cdot \frac{c^2}{4y} \cos \alpha = \frac{c^2}{4y} \sin \alpha$$

integrating

$$\cos \alpha = \frac{c^2 - 2y^2}{c^2}$$

and

$$\sin \alpha = \frac{2y}{c^2} \sqrt{c^2 - y^2}.$$

Substituting this value in the original equation

$$W = [T + ryg(\rho_1 - \rho_2)] \frac{4y}{c^2} \sqrt{c^2 - y^2}.$$

Assuming $T = 35$ dynes, the average value found by method of ripples, and substituting values for the other quantities, $y = 0.3$ cm. (by observation), $g = 980$ dynes, $\rho_1 = 1$, $\rho_2 = 0.8$, $r = 0.02$ cm.

$$W = 48.00 \text{ dynes.}$$

The observed maximum value for W is 44.7 dynes, which justifies approximately the assumption of 35 dynes for the interfacial

tension of water petroleum. A probable explanation of the measurements giving less than 44 dynes is suggested by a consideration of the method of measuring. The rectangular was kept wet with water until petroleum was poured on the water surface. The rectangle was then raised until the interfacial film broke, leaving the rectangle immersed in petroleum and probably partially wet by it. This first measurement was usually high. On again lowering the rectangle and repeating the measurement of the interfacial film, the tension was usually found to be less. Further repetitions gave still lower tensions until a minimum value was reached which was approximately the value obtained by method of ripple waves. These considerations lead strongly to the supposition that a true film was formed after the rectangle was immersed in the petroleum.

Ripple Waves.

The author has devoted some time to a study of the waves themselves and presents here some of the results obtained.

The photograph reproduced in Fig. 4 gives a general idea of the liquid surface when ripples are steadily maintained. The photograph was taken by Vincent's method.[1] The dark cross in the center shows the tuning fork prong and the vibrating plate, the meniscus about the plate making it appear thicker than it really is. The fork and waves are smaller than their real size. The ends of the glass plate act as point sources of disturbance and send out circular waves which interfere and form, as Vincent has pointed out, a family of ellipses and a family of hyperbolas. The author has been able to distinguish these at times by viewing the surface at a favorable angle of incidence. The circular waves interfere also with the straight parallel waves forming a family of parabolas which have their foci at the point source of disturbance. The circular waves in the upper right hand corner were caused by some extraneous vibration of the pan just as the negative was exposed.

Fig. 5 is also a reproduction of a photograph of mercury waves and shows the effect of extraneous disturbances, the pan being placed directly on the floor instead of cushioned as before.

[1] Phil. Mag., 45, 191.

Figs. 6 and 7 are from two photographs taken by means of reflected light, the camera being placed in the position usually occupied by the telescope. A large plano-convex lens of six inches diameter was used to bring the light to a focus at the camera stop. The time of exposure for the photograph of the oil-water waves was nearly ten minutes. The distinctness of the crests furnishes an evidence of the unison of the frequencies of the interrupted light and the waves. The strong damping of waves already mentioned is here shown.

The amplitude of the waves is very small. To get an approximate idea of the size of the waves the author measured the amplitude of the fork which generated the waves. This was done by vibrating the fork more vigorously than usual and observing its motion by means of interrupted light. The frequency of the light flashes was made slightly different from that of the fork so that the latter appeared to move slowly. The magnitude of its excursion was measured by means of a traveling microscope and found to be .0116 cm. Much difficulty was experienced at first in focussing on a definite point of the fork. The device that was finally successful was to fasten a needle to the fork with a bit of wax and observe it through the microscope when the field of the latter was made half dark by pasting a piece of paper over half the aperture. Fig. 11 shows the two positions of the needle when measurements were taken. The reading for position A was taken when the fork was at rest, the position of the needle being accurately shown by the narrow band of light DC. The fork was then vibrated and a reading taken for the position of extreme vibration at A'. The magnitude of the excursion, as already stated, was found to be .0116 cm. The fork's usual amplitude may safely be taken as one-half the amplitude for this case, thus giving the maximum amplitude of the waves generated by the fork as .0058 cm. or $\frac{1}{16}$ of a millimeter.

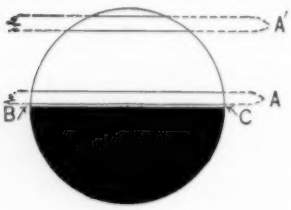


Fig. 11.

The waves die down due to the viscosity of the liquid according to the usual logarithmic decrement. This action of the waves gave

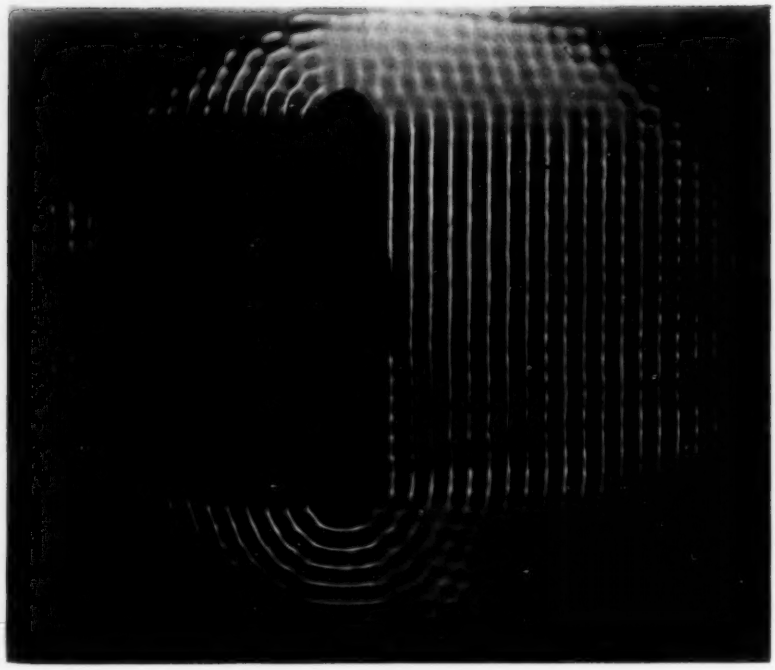


FIG 7.

Photograph of mercury waves.

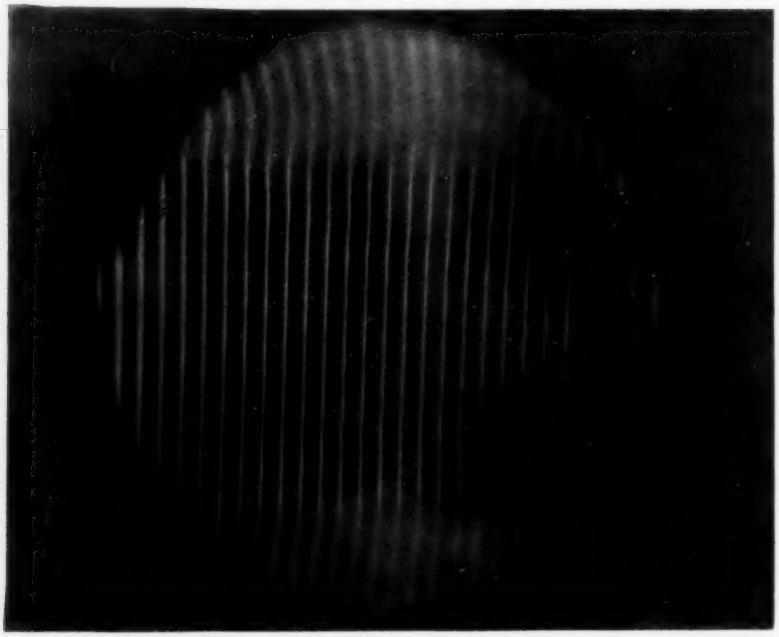


FIG 9.

Waves at the interface of mercury and water.

FIG 8.

Mercury waves.

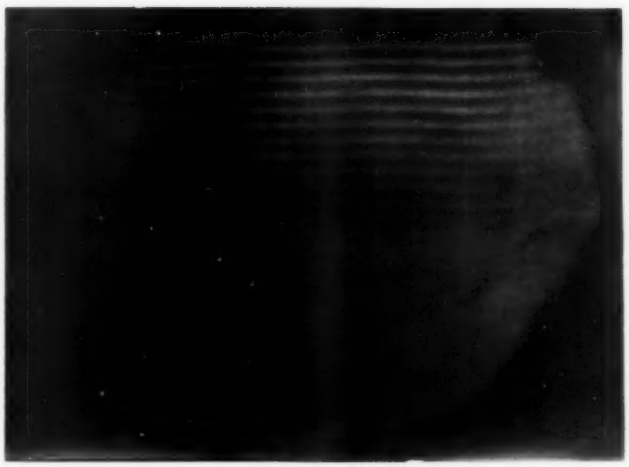


FIG 10.

Interfacial waves, between oil and water.

some trouble in taking measurements of the wave-length, for the telescope would get out of focus as it was moved along. The following diagram (Fig. 12) explains the difficulty more clearly.

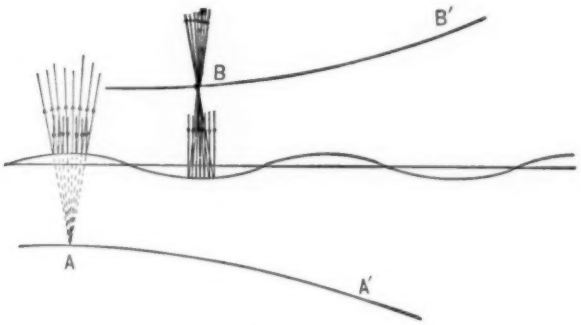


Fig. 12.

The light rays are parallel and, as they strike each crest, are reflected as they would be from a convex mirror, appearing to come from the foci A, A', A'', etc. Similarly, on striking a hollow, the reflection takes place as it would from a concave mirror, bringing the light to a focus at B, B', B'', etc., as only a small part of the reflected light enters the telescope, the waves appear as narrow bands of light with dark spaces between. The waves in dying down become flatter and the foci consequently further removed from the surface of the liquid, so that the telescope which moves parallel to the surface soon loses the sharp focus of the waves. The curves of the foci are shown in Fig. 13.

The surface of the liquid is represented by the x-axis, the vibrating plate being at the origin. The telescope moves along the line $y = 75$ cm. The abscissæ represent distances from the vibrator on the surface, and the ordinates the distances of foci from the surface. The curve for foci of crests is below the surface and for foci of hollows above. The two curves are symmetrical so far as traced. The method of locating the points on the curve was as follows: move the telescope to a position (4, 75) (say) and focus on a crest; turn the telescope away from the surface, keeping the same focus, then bring an object into focus—a small stick of wood was used—and measure the distance from the telescope to the stick. This

distance is the distance from the telescope to the focus. In this way both curves were plotted.

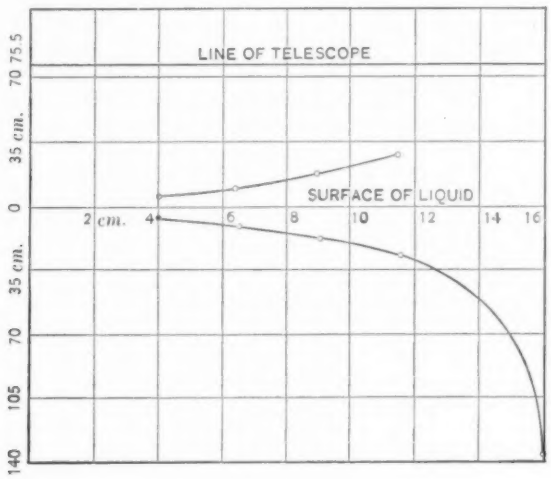


Fig. 13.

In conclusion, the author desires to express his thanks to the members of the Physics Department of the University of California for courtesies shown, and especially to Professor Slate for his advice and encouragement during the beginning of the work. The author is also grateful to Professor Nichols and staff of Cornell University for valuable suggestions given and for the liberal use of apparatus and facilities. Diagrams of apparatus were kindly drawn by Mr. A. V. Saph.

THE PHYSICAL LABORATORY OF CORNELL UNIVERSITY, March, 1901.

AN IMPROVED METHOD OF RATING TUNING FORKS.

By John O. Reed.

ALTHOUGH the tuning fork has long been used as a means of measuring small intervals of time, yet a cheap, convenient and accurate method of rating the fork has remained a thing much to be desired. Of the various methods proposed, the most successful have generally employed the stroboscopic principle in some form or other. Of these the method first described by Michelson[1] is unquestionably the most simple and convenient, although attended with experimental difficulties which materially limit its usefulness and detract from the accuracy of which it is otherwise capable. In the effort to obviate these difficulties the writer has resorted to a number of devices, some of which are, it is thought, sufficiently successful to warrant their description as a distinct method.

In the method as described by Michelson, the standard fork is compared with an auxiliary fork by counting beats; the auxiliary fork is driven electrically and is furnished with a mirror, in which is viewed the flash of a Geissler tube; this tube is actuated by an induction coil whose primary circuit is periodically closed by a pendulum of known period. When the fork is at rest the flashes are seen at one place and appear at rest. When the fork is made to vibrate the flashes form a stroboscopic procession back and forth across the field of view, and from the number of flashes needed to complete the cycle the frequency of the fork is readily determined, provided its value is approximately known.

The difficulty in securing exact periodicity in the flashes of the tube forms a most serious obstacle in the use of the method, as originally described. Michelson points out that if the intervals between the flashes differ by 0.002 of a second, the method becomes

[1] A. A. Michelson, Philos. Magazine, Vol. 15 (5), p. 84, 1883.

impossible for a fork of frequency 128 v. s. He accordingly devotes much attention to the mercury contact in order to make the flashes as nearly isochronous as possible. The sources of error due to this contact are numerous. First, the mercury globule must be exactly in the center of the arc of the pendulum, an adjustment difficult to get and troublesome to maintain. Again if the globule have any appreciable thickness in the direction of the moving point, there is an interval of time between the make and break of the primary current, sufficient to give rise to a system of double flashes of varying intensity. These become extremely confusing to the observer trying to follow the stroboscopic procession across the field of view, and in many cases it is impossible to decide in which direction the flashes are moving. Oscillatory discharges of the coil are also possible, and the flashes become very complex and misleading.

By properly adjusting the resistances in the primary and secondary circuits, and by making the globule thin and flat, the abovementioned difficulties may be largely obviated, but there remain more serious objections; first, the time of discharge of the coil through the tube is not a definite interval, much less is it amenable to control or regulation; again, the mercury globule soon becomes coated with a film of oxide, even when weak currents are used, and this film drags after the contact point, forming a minute tail which prolongs the time of break in some cases as much as one or two thousandths of a second and is always irregular and uncertain in its action. Now, the fundamental assumption in the application of the stroboscopic method is that the intermittent illumination shall be strictly isochronous and that the time of illumination shall be vanishingly small as compared with the period of the motion to be studied, in order that the body may not move sensibly during the illumination. Consequently an illumination that would be entirely satisfactory for a fork of frequency 64, would be inadequate for work upon a fork of frequency 128 or 192, since in the latter case the flashes would be spread out into broad bands of three times the width of the same flashes as seen in the mirror of fork 64.

It is evident, therefore, that a system of illumination intended for use in stroboscopic work must not only be isochronous to any

required degree of accuracy, but if it is to find general application, it must also be entirely under control as regards the duration of the illumination, and this duration must be capable of adjustment within considerable limits. From the above considerations it is clear that the flash of a Geissler tube is but poorly adapted to the stroboscopic study of vibratory motions of even moderate frequency, and a new source of intermittent illumination must be provided. A satisfactory substitute is found in the following arrangement.

A rod R_1, 8 mm. in diameter and 100 cm. long, hung from two short pieces of clock spring, *aa*, carries on its lower end a large lens-shaped mass of lead, L, and four adjustable weights 1, 2, 3, 4, the whole forming a free pendulum of adjustable period. Below the middle of the rod is clamped a ring to which is attached a shortarm l, bearing at its outer end a small needle n. Against this rests a short piece of fine glass tube t, about 0.1 mm. in diameter and attached normally to a vertical axis p which turns in point centers and carries a plane mirror m. As the pendulum swings in a plane normal to the plane of the paper, the mirror is caused to rotate about the vertical axis p, through a relatively large arc whose angular amplitude A, is given in terms of a, the angular amplitude of the pendulum, by the relation $aR = Ar$ or $A = aR/r$, where R and r represent the effective radii of motion of the pendulum and mirror respectively at the point of contact of the tube and needle. The tube is held against the needle by means of a small drop of oil at

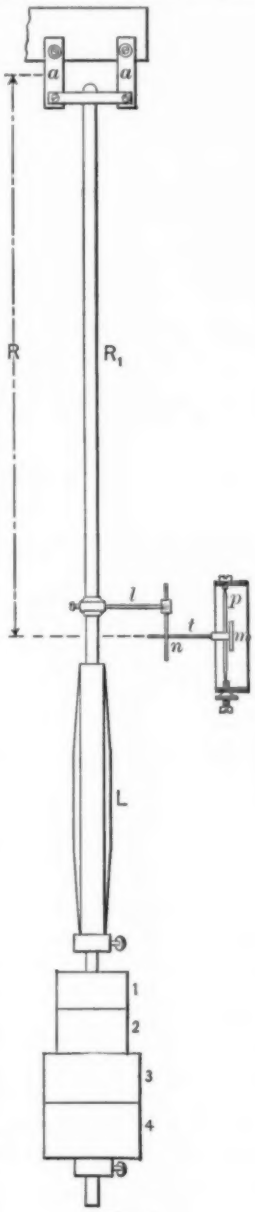


Fig. 1.

the point of contact, the viscosity of the oil being sufficient to carry the mirror on its return swing.

If now there be thrown upon the mirror *m* a beam of light from any convenient source, we have a moving spot of light which may be given any desired linear velocity, and whose successive vibrations are isochronous to a high degree of accuracy. The source of light is a brightly illuminated slit, shown at *s* (Fig. 2) from which the light is focussed at *s'* by the lens *l* after reflection at the mirror *m*. *T* is an Abbé autocollimating telescope, having a totally reflecting prism in the focal plane of the eyepiece, by means of which the light entering the window at *s'* is thrown directly into the axis of the telescope and leaves the objective in

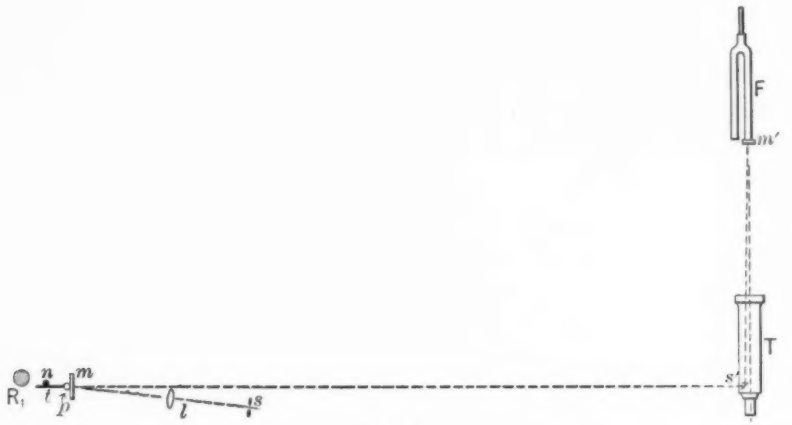


Fig. 2.

parallel rays. Directly in front of the prism is an adjustable slit *s''* (not shown in the figure), thus allowing the telescope to serve both as collimator and observing telescope. The parallel rays from the telescope strike the mirror *m'* normally, and return to form an image of the slit *s''* in the focal plane of the telescope. So long as both pendulum and fork are at rest there is seen in the field of the telescope a single bright image of the slit. If the pendulum be set in motion this image flashes brightly each time the image *s'* sweeps past the small window in the telescope, the duration of the flash with a given pendulum depending upon the width of the window, the width of the image *s'*, the distance *ms'*, and the ratio R/r.

When the fork is set in motion the image in the telescope becomes a stroboscopic procession of images moving across the field, each one so sharp and distinct that coincidences with the cross hair of the telescope may be observed or approximate coincidences estimated to a fifth of the width of a slit image without difficulty.

In practice the free pendulum is rated in terms of the standard clock at the same time that the observations are made on the fork, one observer being able to carry on both sets of readings by working alternately upon first the one and then the other. By determining the period of the pendulum at the time of the observations upon the fork, corrections for effects of temperature upon the pendulum are obviated. The rating is effected by the method of coincidences as employed by Mendenhall [1] in his work upon the determination of g. The coincidences are observed by viewing the flash of a Geissler tube marking seconds, in a system of mirrors, one fixed and the other attached to the free pendulum. The "coincidence" occurs when the composite image of the flash appears as an unbroken line. To this end the pendulum carries a small mirror about 3 cm. wide and 6 cm. long, and near it in the same vertical plane, is placed a fixed mirror of the same size, so adjusted that the image of the capillary part of the Geissler tube appears as one straight piece when seen in the mirrors with the pendulum at rest. It is necessary to take the coincidences with the pendulum of the clock always going in the same direction, in order to avoid the error due to any slight eccentricity in the position of the mercury globule.

Under ordinary circumstances the period of the pendulum can be determined to hundred-thousandths of a second in twenty minutes, usually not more than five coincidences being necessary. During the intervening intervals a sufficient number of cycles of the fork can be counted to give all the data needed to determine its frequency so that the entire process may be completed in half an hour. In laboratory practice it is customary to require the student to make three determinations and take the mean. For this purpose three different pendulums are arranged by making suitable combinations of the adjustable weights 1, 2, 3, 4 shown in Fig. 1, so

[1] T. C. Mendenhall, Am. Jour. of Science, Vol. 43, p. 85, 1892.

that for each pendulum a workable cycle of flashes from the fork may be secured. The best results are obtained by using cycles of not less than five nor more than ten flashes per cycle. The computation is as follows:

Let N be the frequency of the fork to be rated.

Let t_1, t_2, t_3 be the periods of the pendulums 1, 2, 3, where $t_1 < t_2 < t_3$.

Let n_1, n_2, n_3 be, to the nearest integer, the number of complete vibrations made by the fork in times t_1, t_2, t_3.

Let a_1, a_2, a_3 be the number of flashes per cycle made by the fork with pendulums 1, 2, 3; where $5 < a < 10$.

Let c_1, c_2, c_3 be the reciprocals of a_1, a_2, a_3 respectively. Then

$$Nt_1 = n_1 \pm c_1$$
$$Nt_2 = n_2 \pm c_2$$
$$Nt_3 = n_3 \pm c_3.$$

If now $(t_3 - t_1) < \frac{1}{2N}$ then $n_1 = n_2 = n_3 = n$, and we have

$$N = \frac{n \pm c_1}{t_1} = \frac{n \pm c_2}{t_2} = \frac{n \pm c_3}{t_3}.$$

If N be approximately known, *i. e.*, to the nearest integer, then n may be set equal to Nt. If N be wholly unknown the graphical method may be employed to determine its approximate value.

The proper sign to be chosen for c in the three equations is shown from the following consideration: The free pendulum as it swings decreases in period very slightly as the amplitude decreases; this decrease in period is evidenced by a slight but unmistakable variation in a, the number of flashes per cycle in each individual case, and consequently by a corresponding variation in c, the fraction of a swing by which the fork exceeds or falls short of an integral number of swings in time t. Now as t decreases this excess will diminish or the deficit will increase; therefore if a is found to increase during the experiment c is positive; if a is found to decrease, c is negative.

In making a determination the apparatus is first carefully adjusted until the spot of light falls exactly on the window of the

telescope when the pendulum is at rest, and the image of the slit is very bright. The pendulum is then started, the fork set in vibration and the cycle of flashes roughly determined. If necessary the pendulum must be adjusted by moving the small weights at the lower end until the number of flashes per cycle lies between 5 and 10. For convenience of reference a small sketch of the pendulum showing position of weights is usually appended. The pendulum observations are then started by taking three or four coincidences in quick succession, to get its cycle roughly, after which the coincidences may be taken in groups of three at intervals of five minutes or more as the work proceeds. Only one of each group is used, the others being merely to insure against possible failure to keep track of the pendulum. In the intervals between the observations on the pendulum the fork is bowed and the coincidence of the flash with the cross wire of the telescope awaited; when this occurs the flashes are counted continuously, the observer making note of the coincidences, underscoring those especially sharp, as long as the fork will run, or until the images can no longer be seen sharply separated. The fork is bowed again and the observations repeated. In the computation the mean value of a is to be taken, and exceptionally sharp coincidences are to be assigned greater relative importance in determining the value of a; from this the value of c is computed.

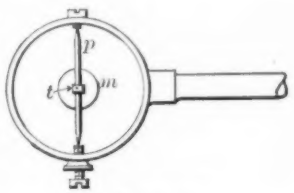


Fig. 3.

It is well to note the time of beginning each set of observations upon the fork, as in this way the varying value of a may be readily followed.

The following example of a complete determination will render the method clear:

Pendulum gaining. 6 *in* 64 *; better* 14 *in* 150. *January* 2, 1898.

Transits.	Seconds.	Vibrations.	Period.	Fork bowed.
3 02 11				6<a<7
03 15	64	70	0.914	nearer 6.
05 45				
07 11	300	328	0.9146	*3, 07, 11*
14 21				6, 44, 88
15 25				
16 51	880	962	0.91476	6, 44, *63*, 88
29 01	1610	1760	0.91477	
30 27				6, 25, 44, 88
31 31				....
35 49	2018	2206	0.914778	*3, 16, 51*
38 19				6, *31*, 43,
39 45	2254	2464	0.914773	86
43 41				6, *37, 74*
45 07	2676	2816	0.914773	6, *43*, 49, 98

$t_1 = 0.91477$ sec.

Computation of a_1.

Flashes.	Cycles.	a.		
1st set. 44	7	6.285	$6.285 \times 9 =$	56.565
63	10	6.300		6.300
25	4	6.25		6.250
			11)	69.115
			$a =$	6.2_83
2d set. 31	5	6.200		6.200
43	7	6.1428	$6.1428 \times 4 =$	24.5712
49	8	6.125	$6.125 \times 3 =$	18.375
37	6	6.166	$6.1666 \times 3 =$	18.4998
			11)	67.6460
			$a =$	6.1487
			Mean $a_1 =$	6.2158

a_1 decreasing, hence c_1 is negative.

$c_1 = -\ 0.16089$

$n = 30.$

$t_1 = 0.91477$ sec.

$$N = \frac{29.83911}{0.91477} = 32.619.$$

A second fork gave for ten different determinations, by different persons on different days, the following values:

Temperature.	N_t	N_{20}
13.0°	32.493	32.468
17.5°	32.475	32.466
17.7°	32.473	32.465
18.8°	32.472	32.468
19.5°	32.469	32.467
21.0°	32.464	32.467
22.7°	32.458	32.468
23.0°	32.457	32.469
23.0°	32.455	32.467
23.4°	32.456	32.469

Reduction to 20° C. has been made by the use of Koenig's formula,[1] $\triangle N = \frac{-N}{8943}$ per degree centigrade.

For forks of higher frequency the requirements are greater accuracy in the adjustment of the light upon the telescope slit when the pendulum is at rest, more powerful illumination of the slit and greater velocity of the moving image. The last requirement is met by increasing the ratio R/r, and the distance ms'. With a slit illuminated by an acetylene flame or an arc light, no difficulty is experienced in reducing the duration of the flash to 0.00001 of a second, a period sufficiently short to show the slit image in the telescope, sharp and clear, when reflected from the mirror of a fork making 256 double vibrations per second, even in the middle of its swing. This is readily seen from the following considerations: Assume the amplitude of the motion of the slit image as seen in the telescope to be 5 mm.; then the maximum velocity of the image is 8,050 mm. per second. Also assuming the limit of the resolving power of the eye to be 0.073 mm. at the distance of distinct vision, we have for the duration of the flash

$$t = \frac{0.073}{8050} = 0.000009 \text{ second.}$$

In case it is not desirable to attach a mirror to the fork under investigation, Michelson's second method may be applied; that is the flashes from the pendulum may be received directly into the objec-

[1] Koenig, Wied. Annalen, 1880, N. F., 9, p. 413.

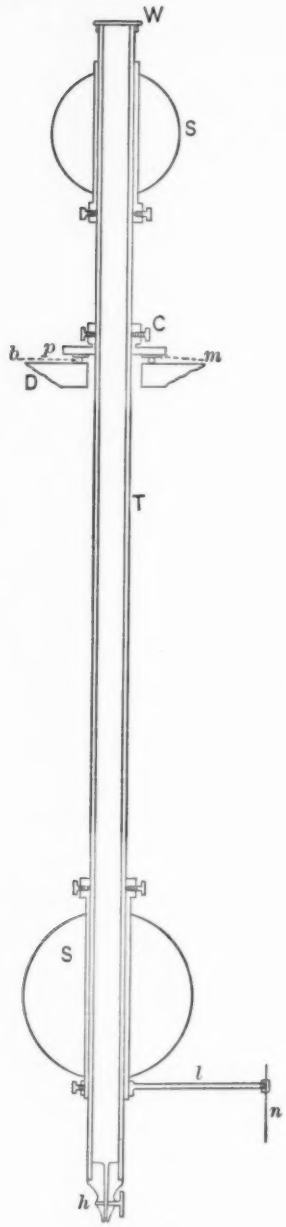


Fig. 4.

tive of a low power microscope in front of which is placed the fork with one prong sharply focussed so as to fill half the field when at rest. The flashes will then give a stroboscopic procession of shadows instead of images of the slit as before. Even by this method forks may be rated, whose frequency would entirely preclude the use of the Geissler tube.

For the purposes of an investigation of which an account will appear later, it became necessary to vary the period of the flashes continuously and by any desired amount. To this end a special form of free pendulum was devised, the main features of which are shown in Fig. 4.[1] A piece of bicycle tubing *T*, 2.5 cm. in diameter and 100 cm. long is fitted with an adjustable collar *C*, to which is attached a brass plate *p*, 7 mm. thick. On the under side of this plate are fastened two small plates *mm'* of hardened steel, each 3 mm. thick and having their under surfaces ground plane. These plates rest upon two small bicycle balls $\frac{3}{32}$ of an inch in diameter, which in turn rest in two small depressions in the bed plate *D*. At either end of the tube *T* are clamped spheres of lead *SS'* of 7 and 10 kilograms mass respectively. By means of set screws these spheres may be given any desired position on the tube.

At the lower end of the tube is fitted a steel plug and stopcock *h*, by means

[1] This pendulum was made by Mr. R. H. Miller, instrument maker to the university, to whose practical suggestions and skillful workmanship much of its admirable performance is due.

of which a quantity of mercury held in the tube may be varied at will. A small collar holds the horizontal arm and the needle for actuating the rotating mirror as shown in Fig. 1. The bed plate *D* rests upon three short pointed feet which are held in place upon a steel plate on the supporting frame by the familiar hole and slot arrangement. The whole is supported by a rigid frame of suitable height, firmly clamped to a stone pier.

By adjusting the relative positions of the supporting plate and the two lead spheres, the period of the pendulum may be varied within wide limits. In practice the period is kept approximately either 0.5 second or one second for a single swing. In the latter case the system with rotating mirror attached will vibrate about three hours and still retain sufficient amplitude to permit of satisfactory observations. With smaller balls the duration of the motion may be made even longer.

As will readily be seen the variation of the period in order to obtain a workable cycle of flashes may now be made to any degree of accuracy and *also while the pendulum is in motion.* To this end mercury is poured into the tube until the period is slightly longer than desired. The apparatus being adjusted the pendulum is set in motion and the cycle observed. If not satisfactory a tap of the pencil upon the stopcock *h*, allows a fine stream of mercury to flow out into a convenient receptacle, and the flashes are observed until a workable cycle is secured when a second tap of the pencil closes the stopcock. In this way the pendulum may be "*tuned*" to any desired period with great accuracy. The enclosed column of mercury serves the additional purpose of rendering the pendulum much less sensitive to changes of temperature, and in case it were deemed desirable, practically complete compensation could be secured in this way. In such case the variation of the period may be effected by placing small weights upon the small cap *w*, at the upper end of the pendulum.

By the use of the above pendulum the following values were sesured for the frequency of a fork marked 512 v. s. using the mirror and flash method.

Temperature.	N_t	N_{20}
20.2°	256.019	256.025
21.0°	255.993	256.022
22.3°	255.960	256.026
20.8°	255.995	256.018
23.0°	255.934	256.020

From the above values it is evident that the method is applicable through quite a range of frequency. In as much as the correction for temperature increases as the frequency increases, it is evident also that if any accurate results are to be obtained with forks of high frequency, the question of the exact temperature of the fork and the means of keeping the temperature constant become fundamentally important. For a fork of frequency 256, the temperature correction is 0.0286 per degree Centigrade; hence an error of 0.3° C. in the temperature of the fork would account for the maximum outstanding difference in the series of values given above. Under ordinary circumstances the temperature of the fork can best be determined by attaching it to some large mass of metal, with which the thermometers may be brought into thermal contact either by means of mercury cups or by packing the bulbs in tin foil. By enclosing the fork in a hollow cylinder of metal with walls some centimeters thick it is admirably shielded from external temperature effects and its exact temperature may be very accurately determined. To the problem of rating a fork so thermally shielded the method lends itself most readily; only a single tube closed by glass plates being needed for the ingress and egress of the beam of light. The results thus far obtained seem to justify the assumption that with a fork so shielded the method will give the frequency constant in third place of decimals.

For convenience of reference a partial bibliography of the subject is appended.

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University of Michigan, Ann Arbor, March, 1901.

GEORGE FRANCIS FITZGERALD.

BY J. LARMOR.

THE eminent scientific man whose work we here propose to review was born in 1851, second son of William FitzGerald, a distinguished theologian and metaphysician, Bishop of Cork, and afterwards of Killaloe. His early education was conducted at home, in company with his brothers and sisters, under the tuition of a lady who was for many years a valued member of the household, a sister of the great mathematician and logician George Boole, who was professor in the Queen's College at Cork. He was good at all subjects requiring close observation from his earliest years. On passing to Trinity College, Dublin, at sixteen years of age, he soon made his way to the top in the classes of his year, obtaining his degree in 1871 at the head of the lists both in Mathematical and in Experimental Science. He became expert at gymnastic exercises and was for many years a fine racquet player; he also took a considerable part in the literary and social clubs of the College. His athletic skill was turned to account a few years ago in experimenting on the mechanics of flight.

The years from 1871 to 1877 were spent in reading with a view to the examination for Fellowship: vacancies were at that time very rare, but he was successful on his second time of trying. The course of study contained portions of the classical writings of Lagrange and Laplace, Hamilton and MacCullagh; and the close scrutiny of the modes of thought of masters such as these doubtless formed an admirable preparation for his subsequent scientific career. During this period he also made a formal study of metaphysics; he was much attracted by the idealism of his countryman, Bishop Berkeley, which kept cropping up in his discourses as the connecting link between material science and higher things, up to the end of his life. It formed one of the manifestations of a poetic and deeply religious temperament.

For some years after obtaining Fellowship, he served with great success as a College tutor ; his side was very popular, so that it was always full a long time in advance ; and his services were in great demand as a director of the social and athletic organizations of the College. He gave up tutorial work on being elected in 1881 to the Chair of Natural and Experimental Philosophy, which he retained until his death last February. The care of a laboratory not adequately endowed, and a large amount of teaching of elementary classes, took up most of his time ; what remained was much broken up in acting as Examiner in Physics in other Universities, and as referee of scientific memoirs for learned Societies, in copious correspondence with scientific friends who were sure of obtaining acute and stimulating opinions on all knotty points submitted to him, and in the discharge of public duties which of late years had been increasing largely in extent and importance. Yet he found time to become one of the most learned men of the day in all branches of physical science, both theoretical and applied, and to leave his mark on the scientific progress of his age.

The physical problem attacked in his first published paper (*Roy. Soc. Proc.*, 1876) was the then recently announced Kerr effect, the displacement of the plane of polarization of light on reflection from a magnet. By an argument based on general considerations, he connects this phenomenon with the double refraction indicated by the different velocities of propagation of right- and left-handed circular vibrations in magnetized media, as Stokes and Kelvin had already independently suggested. The details of the argument are corrected in the papers next following ; but by a rough numerical estimate he shows, as Kundt experimentally verified some years after, that the Faraday rotatory power in traversing a film of magnetized iron must be very great, and that the effect similar to Kerr's which must exist on reflection from a naturally rotatory substance such as quartz would be actually inappreciable. By neat and simple appliances he repeats Kerr's experiment : he moreover finds that the polarization really becomes elliptic, so that there is a phase-difference involved in addition to the rotation : in further confirmation that the effect is due to the magnetized substance and not to the field he finds that it is entirely cut off when a film of gold-leaf is spread over the reflecting

pole of the magnet. At first he thought that this elliptic polarization was directly involved in the double refraction; but in the next memoir he recognized that the difference of phase must arise, like the elliptic polarization of ordinary metallic reflection, "from the efficient reflecting surface being of some depth."

The subject of the connection of light with magnetism was thus opened out, as a necessary preliminary to a more definite treatment of this problem of magnetic reflection; and, being led thereby into Maxwell's electric theory, he follows it up two years later in a memoir "On the electro-magnetic theory of the reflection and refraction of Light" (*Phil. Trans.*, 1880, pp: 691–711) presented to the Royal Society at the end of 1878, which constitutes his most complete formal treatise. Maxwell in his memoirs on electrodynamics, and particularly in the definitive exposition published in *Phil. Trans.*, 1865, had shown that the hypothesis that light consists of electric waves gives a straightforward account of the mode of its propagation in crystals, and one which is free from the difficulties attaching to the existing mechanical theories; yet he had not made any explicit application to the dynamical laws of reflection. It was left for Helmholtz to make the remark in 1870, in a footnote to his first formal memoir on the theory of electrodynamics (*Gesamm: Abhandl:* i, p. 558) that in this respect also Maxwell's theory leads directly to the observed results by supplying the appropriate boundary conditions at the transition between the official media, whereas it had proved to be impossible legitimately to adjust any definite mechanical theory so as to admit those conditions. This statement was confirmed in a detailed formal investigation published by H. A. Lorentz in 1877. The subject of reflection is not noticed by Maxwell either in the *Phil. Trans.* memoir of 1865 or in his treatise which appeared in 1873, two years after the date of Helmholtz's memoir. It seems possible, from various incidental remarks, that he was then content with the evidence afforded by the correspondence of the velocity of electric propagation in a vacuum with that of light; and that numerical discrepancies, such as that between the index of refraction in transparent bodies and the value of the square root of the specific inductive capacity, and the fact that metallic sheets are less opaque than their conductivities ap-

peared to demand, suggested that a closer view of the vibrations of the material molecules and their interaction with the æther would have to be acquired before the dynamical problem of optical reflection became ripe for definite treatment. He was in fact in full possession as early as 1869, as Lord Rayleigh has recently shown, of the manner in which the phenomena of normal and anomalous dispersion are dominated by the influence of sympathetic vibrations in the molecules of the material medium. The analysis of this problem of optical reflection that was now independently developed by FitzGerald is of high import from the point of view of general electrodynamic theory. He first announces the proposition (most pointedly in the abstract of his memoir, *Roy. Soc. Proc.*, 1879) that by introducing a vector (ξ, η, ζ) representing the magnitude of the total disturbance of the element of æther (of whatever nature it may really be) of which the magnetic force constitutes the time-rate of growth or velocity, the electrodynamic scheme of Maxwell, for dielectric media of aeolotropic quality however general, can be immediately reduced to the standard form required by the principles of analytical dynamics; namely, the effective kinetic energy T and the potential energy W of the medium being given by the expressions [1]

$$T = \frac{\mu}{8\pi} \int (\dot{\xi}^2 + \dot{\eta}^2 + \dot{\zeta}^2)\, d\tau$$

$$W = \tfrac{1}{2} \int U d\tau,$$

where $\partial\tau$ is an element of volume and U is a quadratic function of (f, g, h) characteristic of the material medium, (f, g, h) representing the curl of (ξ, η, ζ), or

$$\left(\frac{d\zeta}{dy} - \frac{d\eta}{dz}, \frac{d\xi}{dz} - \frac{d\zeta}{dx}, \frac{d\eta}{dx} - \frac{d\xi}{dy} \right),$$

then the equations of electromotive propagation are obtained in the Lagrangian manner by annuling the variation of the action, that is from the *single relation*

$$\partial \int (T - W)\, dt = 0.$$

There are, however, other analytical expressions for T and W

[1] In case the medium is not homogeneous μ would be inside the integral.

which would lead to the same equations of bodily propagation, as witness the various elastic mechanical theories of light which give the correct laws of transmission; in fact by integration by parts we may express $T - W$ in the form of a new volume integral together with a surface integral extended over the boundary of the region considered, and of these only the former part is concerned in the formation of equations of propagation, though the latter is essential to the relation connecting stress and strain.

FitzGerald points out, following his countryman MacCullagh, and also Green in a different connection, that not merely the equations of bodily propagation, but the equations of transition expressing continuity of stress and displacement between the media at the reflecting interface, are definitely and consistently involved in this way of formulating the problem by a single equation, the surface integrals in the reduced variation of the action determining the latter relations. He then remarks that the electric theory, as thus expressed in terms of this magnetic displacement (ξ, η, ζ), is formally identical with MacCullagh's dynamical theory of crystalline reflection and refraction published thirty years before. The problem which MacCullagh then proposed to himself was to work backward from an analysis of the actual circumstances of crystalline reflection, as determined mainly through the experiments of Brewster and Seebeck, to a type of Lagrangian functions $T - W$ for the media of propagation which would correspond in their consequences with these very various test phenomena. It has always been admitted that he was entirely successful in this analytical quest: but he definitely proclaimed his inability to conceive of any kind of material elastic medium, as ordinarily understood, whose properties would be represented by the relations finally obtained. The fact that in any material medium, in which dynamical phenomena are determined entirely by interaction between internal stress and forces of the usual non-polar kind acting from a distance, there would on MacCullagh's scheme be an unbalanced torque on each element of volume, shows that a mechanical theory of this type can subsist only if there are polar forces (*e.g.*, *quasi*-magnetic) capable of compensating the torque, or if there is a kinetic reacting torque arising from gyrostatic rotations forming a part of the constitution of the medium. In later years

Lord Kelvin showed how to construct a cellular material system with gyrostats mounted in its elements, which would form an actual model of this rotationally elastic medium of MacCullagh. The hasty assumption that the analysis of the latter must necessarily be identical with the tentative mechanical theory of F. E. Neumann, because it gave the same results, had led to the real significance of MacCullagh's investigation being overlooked at the time; doubtless too it was felt the possibilities of purely mechanical media had not been exhausted. It was accordingly an advance of prime importance when FitzGerald pointed out in 1878 that the strain, of unknown nature but known relations, which MacCullagh's theory required, was precisely the one, equally unknown to our perceptions of sense, on which Maxwell's electro-optic theory is based; his memoir in facts terminates with the remark that Maxwell's theory, if it "induced us to emancipate our minds from the thraldom of a material ether, might possibly lead to most important results in the theoretic interpretation of nature."

Other considerations may be mentioned in illustration of the significance of FitzGerald's mode of considering the subject. In the statement of electrodynamic theory elaborated by Heaviside and by Hertz, which has been widely accepted as a powerful aid to concise formulation and development, the main feature has been the elimination of the potential functions of the vector electric current and the scalar electric distribution, that were introduced in the course of Maxwell's analysis: in FitzGerald's dynamical procedure, from the basis of the principle of Least Action, no such potentials occur. Indeed Maxwell had himself pointed out (*Phil. Trans.*, 1868) that the equations of electric propagation were most compactly expressed (when their theoretical basis was not in question) by the pair of circuital relations between the fundamental vectors which result from the elimination of these potentials. In FitzGerald's analyses the relation in which the interfacial conditions stand to these bodily relations is moreover directly involved. The problem as treated by him was however limited expressly to the case of transparent media, in which there are no wandering electric charges. The natural way to extend his method to the more general questions in which charges are in finite translatory motion, either from conduction or by convec-

tion of the material system, is to retain his specification of $T-W$ for the free æther, but to introduce into the process of variation the conditions imposed by the presence of these electrons ; in the reduction of this more general problem to differential analysis suitable for material media treated as continuous, the employment of Maxwell's potentials can hardly be evaded.

The other general remark is that, in the question as to whether Maxwell's electrodynamic scheme is sufficiently wide to cover the facts, it would have been possible to reason from the optical problem of crystalline propagation already fully analyzed by Green, Neumann, and MacCullagh, in the light of the very various and easily executed optical experiments, to the more general electric problem in which crucial experiment was for a long time almost unattainably difficult. There was fair ground for the inference from MacCullagh's analysis that his scheme was the unique one of dynamical type that could lead to the laws of crystalline reflection : no demonstration of this was indeed given, but the variety of experimental laws to which the solution had to conform created a presumption almost equivalent to proof. That scheme might be made more general by assumption of effective compressibility of the medium, *i. e.*, convergence of the magnetic vector in the electric interpretation, thereby introducing compressile waves ; these would however travel by themselves in entire independence of the transverse waves. But a scheme such as Helmholtz's, which introduced to any sensible extent compressile waves interacting with the transverse ones, would in so far come into conflict with the optical evidence. This question whether Helmholtz's proposed generalization of Maxwell's theory had any actual basis remained a main problem of theoretical electrodynamics until the discovery of electric radiation by Hertz : it has now been tacitly dropped, not so much on account of any specific experiment in disproof, as through the conviction, created by the ever-widening range of correspondence between electric and optical radiations, that these are the same thing at bottom. Towards this conviction the illustrative experiments of FitzGerald and his pupils had their share in contributing. It does not appear that he ever paid much attention to the Helmholtz theory : it reposed really on ideas involving action at a distance, and from the optical point of view which we

have here attempted to sketch there was no need for its wider generality.

The extension of this theory of optical reflection to the magneto-optic problem was effected by FitzGerald by introducing, after Maxwell, terms into the function $T - W$ so as to represent interaction between the imposed magnetic field and the optical vibrations in the medium. He finds, however, that the interfacial conditions in the dynamical problem could not now be satisfied by means of transverse waves alone: and, reserving the theoretical difficulty thence arising, he completes the analysis without introducing compressile waves by neglecting the condition involving the normal disturbance, as Fresnel had done under similar circumstances. This gives him a magneto-optic effect resembling in details, as regards intensity, the description published by Kerr in the same year, but without changes of phase.

Finally it is pointed out that this is only a first approximation to an exact theory, on account of the metallic character of the reflector being ignored, and of the difficulty above mentioned—the question of phase in particular awaiting a more comprehensive theory in which the relation of matter to æther would be more exactly expressed. The difficulty has since been removed by considering the effect of the imposed magnetic field to be a structural modification in the material medium, rather than a fundamental change in the dynamical relations between the electric vectors; and FitzGerald's analytical work still remains applicable. The extension of the metallic media is now formally made, as Ohm's law indicates, by taking the refractive index to be a complex quantity: with this generalization the analysis has been extended by various writers including Lorentz, Goldhammer, and Drude, but most completely by Leathem[1] and by Wind,[2] and shown to embrace satisfactorily all the mass of detail that has been brought out in recent years in experimental magneto-optic investigations.

While these developments were proceeding, FitzGerald was also taking a close interest, stimulated in part by his uncle, Dr. Johnstone Stoney, in the theory of the Crookes radiometer, then coming

[1] Phil. Trans., 1897.

[2] Archives néerlandaises, 1897; or PHYSICAL REVIEW, Vol. VI.

into shape; this he studied in its connection with the cognate subject of the constitution of the gaseous layer which supports a liquid in the spheroidal state without contact with the hot slab, and the state of stress that is as a consequence involved in it. In an elegant subsidiary investigation (March, 1888), of the state of internal pressure in a gas, arising from supposed presence of some coordination in the directions of motion of the molecules, he finds that when the directions are simply related to a single axis an internal stress of the Maxwell electrostatic type results; in the general case the stress is not specially conditioned. This result is to be expected when we reflect that it can be proved that the Maxwell type of stress is the only one, symmetrical with respect to an axis, that can maintain itself in equilibrium. In FitzGerald's point of view the normal pressure on the warm side of a vane of a radiometer may differ from that on a perpendicular cold side because the stress in the gaseous medium is not isotropic; how far this mode of representation is consistent with the more complete analysis of internal stress in gases due to temperature-gradient, by O. Reynolds and by Maxwell, has been a subject of controversy.

In 1881 a new departure was initiated in theoretical electrodynamics by J. J. Thomson, in an investigation with the aim of determining directly the effects produced by moving charged bodies, by means of Maxwell's equations of the electric field combined with the appropriate conditions at the surfaces of the moving conductors. The question is whether the hypothesis that the moving charge acts after the manner of a current-element is an independent assumption, or how far it is already implied in Maxwell's scheme of the electric field. Neglecting the self-induction of the æthereal displacement-currents surrounding a moving charged sphere, that is, assuming instantaneous transmission of effects, it was shown that the continual adjustment of the electric force in the field, so as to be normal to the conducting surface as it moved through space, entailed a magnetic field of the type demanded. Thus if the charged body carries its electric field along with it, the same as if it were at rest, that convection creates the magnetic field aforesaid. At speeds comparable with the velocity of radiation, the result of taking account of this finite velocity of the electrodynamic transmission is that the

electric field thus convected, when the steady state of motion has become fully established, is somewhat flattened equatorially, and the magnetic field is correspondingly changed : but ordinary electrodynamics based on the convection of electrons can safely leave such extreme cases aside. An important result in this paper was the remark that this field of magnetic (kinetic) energy carried along by an electron involves an addition to its effective mass ; the question has in fact more recently been propounded whether all inertia of matter may not be of cognate type. The paper took its rise from the magnetic deflection of the cathode stream in a vacuum tube, as described by Crookes ; and, when some of the analytical computations had been corrected by Heaviside, this memoir of Thomson's constituted the beginnings of what has now become the basis of electrodynamic theory. It is interesting also to observe in it the suggestion that the nearly abrupt changes in the magnetic field arising from the sudden stoppage of the cathode particles may be responsible for the phosphorescence excited by them in the glass: how much more they are actually responsible for was revealed years after through an accidental observation of Röntgen's. In a critical paper FitzGerald had something to add : he pointed out that as the spherical conductor of finite size, to which the analysis related, moved along, the field of æthereal displacement is suppressed in the space that the sphere comes to occupy and is instantaneously established in the space it leaves behind : these changes constitute impulsive displacement-currents : when their vector potential is added to that of the displacement-currents of the field, the result does not give the correct magnetic force—it gives in fact none at all—the current of convection of each superficial element of the charge on the sphere has to be included as well. Thomson, without explicitly including the latter, had done what comes to the same thing by addition of a term required to make the vector potential circuital, thereby implying that the æthereal current must be supplemented so as to make it everywhere circuital also. Thus on closer examination, each portion of the electric charge is found to act independently, and so far from our being able to exclude the electric charge from view by merging it in interfacial conditions, it turns out that its convection is the very cause of the phenomena of the electric

field. When considered from the point of view of the æther, the hypothesis that the total current is circuital, which lies at the very root of Maxwell's theory, involves and is equivalent to the magnetic influence of moving charges, which was verified experimentally before this time by Rowland but has recently again been challenged on experimental grounds. Following out the same train of thought, FitzGerald soon after introduced order into an obscure passage in Maxwell's exposition by inserting the term representing the current due to the convection of charged bodies, which had been overlooked.

In 1882 he contributed three papers to the Royal Dublin Society on the conditions necessary to excite electric radiation; at first he was led into inconsistencies by some confusion between the true exciting cause of the radiation existing in the radiating matter alone and the displacement currents in the surrounding æther to which Maxwell formally ascribes like properties. But he gradually gained firm footing and began to think of the vector potential as something *propagated in time* from the true electric sources, yet remaining essentially the same function which Maxwell treated as an *instantaneous* potential of the true currents and the fictitious æthereal currents taken together. He concludes that the production of radiation by electrical means is possible, that "the interactions between the molecules of matter and the æther are of the same character as the electromagnetic actions with which we are acquainted," and at this early date he goes on to express the opinion that the discharge of a condenser arranged so as to have very little inductance in the discharging circuit may be made to send out actual electric radiation of manageable wave-length.

Following up this train of ideas he published towards the end of 1883 an investigation of the intensity of radiation that would be electrically obtainable. His radiator was an alternating electric current traversing a small circuit, or, as he afterwards described it, a vibrating magnetic doublet, in contrast with the later electric doublets of Hertz. In this brief, but for the time very remarkable paper, he starts with an expression for the vector potential A as *propagated* from the true current $i_0 f(t)$ in the linear circuit, namely the vector formula:

$$A = \mu i_0 \int \frac{f(t - r/c)}{r} ds;$$

then calculating the kinetic energy by the formula

$$T = \tfrac{1}{2} \int (Fu + Gv + Hw) d\tau,$$

where (u, v, w) is now total current, and $\partial\tau$ an element of volume, he finds that in the spherical shell between radii R and $R + \partial R$, this involves an amount of energy which is proportional to ∂R simply and another amount which also varies as R^{-2}. The latter part he identifies as belonging to the "forced displacement-currents" around the vibrator, which start the radiated displacement-currents of which the former term represents the energy. The energy thus radiated per second comes out as

$$\tfrac{8}{3} \pi^4 (i_0 A)^2 \frac{\mu}{c^3 T},$$

where A is the area of circuit, T the period of the simple harmonic alternations of the current, and c the velocity of light. He points out that all of this formula except the numerical factor could have been obtained from the theory of dimensions alone, assuming of course the existence of clear ideas of the process involved. This radiated energy is very small in amount except for high frequencies; with 10^7 alterations of the current per second it amounts to $\frac{1}{10} (i_0 A)^2$ ergs per second. He suggests the possibility of estimating the magnetic moment of a gaseous molecule from the radiating power of the gas. He had here not quite got to the conception of identifying the source of radiation with the motions of the electrons in the molecule; Hertz afterwards approached closer when he took for his theoretical vibrator an alternating electric doublet instead of a magnetic one.

As everybody knows, the apotheosis of these studies of radiation, so clearly presented to the mental eye, came three years later in Hertz's detection of the phenomena which have now, in wireless telegraphs, become developed into a commonplace of civilization. Beyond all question the genius of Hertz deserved this success and he stands alone. But the tracing of the history of scientific thought

on the eve of a great advance is always fascinating and instructive. At the very time FitzGerald was formulating these profound yet direct views, or soon after, Oliver Lodge, his close friend and in a sense his pupil, was actually engaged in feeling out the nodes of the electric waves that ran along his discharge-circuits like sound-waves along pipes, and there can be little doubt that the goal of the experimental demonstration of Maxwell's theory would have been reached along this path, though not perhaps with the same directness as Hertz attained. Indeed it appeared that phenomena of the same kind as Hertz's had been noticed long before by Joseph Henry, and had suggested thoughts of propagation of electric influences; now when we are instructed, the difficulty is so to speak to avoid them.

Referring again to this paper on the intensity of radiation, we see clearly emerging the conception of an electric source, and of the electric field as formulated in terms of vector and scalar potentials directly propagated from the sources. This is the idea which H. A. Lorentz developed into a general theory of electrodynamics in 1892, ten years afterwards. Its expression has attained to ultimate simplicity in the compact statement by Levi-Civita in 1897, that vector potential, varying as usual inversely as the distance, travels in time, with the velocity of radiation, from each element of true current, and static electric potential similarly travels from each element of electric charge, and that the electric force (on electricity at rest) and the magnetic force in the æther are expressed in terms of these potentials in Maxwell's manner. But this kind of vector potential is a difficult one for purposes of analysis of media treated as continuous, its value at a point at a distance r from a source depending on the strength of the source not at that instant but at a previous time—previous by r/c when the source is at rest. In practice Maxwell's original potentials are much more convenient: with him there is no finite propagation of potentials, their values depending on the strengths of the sources at the very instant considered, but to attain to this simplicity the change of so-called æthereal displacement must be treated as current in Maxwell's manner, as well as the convection of the charges; it is only for the total current thus constituted that the simple laws of the early electro-

dynamics of Ampère and Faraday are strictly applicable. In 1893 the formal development of electro-dynamic theory from this point of view, inspired at the start by Fitzgerald's electric interpretation of MacCullagh's optical analysis, and by mechanical models based on Lord Kelvin's construction for a rotationally elastic medium, and resting directly on the broad dynamical basis of the principle of Least Action, was initiated in Great Britain, and was practically complete early in 1897, running parallel in the main results, though not in method or mode of development, with Lorentz's final presentation in his tract of date 1895. This mode of investigation immediately obtained the strong and active support of FitzGerald: the discovery by Zeeman and Lorentz in 1897 of the subdivision of the lines in a spectrum by a magnetic field afterwards provided the stimulus required to attract general attention to a theory whose logical processes were difficult and unfamiliar; so that this point of view is now universally prevalent, and is perhaps occasionally pushed to extremes that would have discredited it when in the purely theoretical stage. The distinction between the analytical potentials of Maxwell and the potentials that are actually propagated from true electric sources had been, as may be gathered from the account here given, fermenting in FitzGerald's mind, though it is only recently that it has emerged into the simplicity demanded for its general recognition: it appears from one of his last letters that it had been his intention to work the subject up, if time had allowed, into a paper for the British Association last September.

FitzGerald's skill in intuitional construction was illustrated by a striking model of the functions of the æther, invented in 1885 in connection with the dynamical theory of his memoir on optical reflection. The problem was to form a simple representation of a medium possessing the rotational elasticity above described. He effected this, as regards two dimensions of space, by mounting a number of wheels freely on a board, and connecting adjacent ones in pairs in various ways by elastic bands stretched over them, so that, as the bands maintained a hold, the wheels became bound together into a single elastic system. Fixity of the centres of the wheels implied incompressibility of the medium. A rotation imparted to one of the wheels spread itself to the surrounding ones and at the same

time set up elastic stress in the bands: this stress represented a field of electric or æthereal polarity—the poles being the two sides of a band, one extended the other shortened. A rotational disturbance imposed at any place in such a medium was propagated with finite speed on account of the finite moments of inertia of the wheels: the kinetic energy of the revolving wheels represented the magnetic energy of the field, which was thus a direct consequence of the changing electric strain. In the state of equilibrium free from polarization the elastic bands were supposed to be free from strain. A region in which the wheels were loose represented a conductor. An intrinsic polarity may be introduced into this medium at any place by rotating one or more of the wheels there situated, making them slip underneath the bands that pass around them: when they are released there will be a permanent field of strain, *i. e.*, an electric field. The lines of electric polarity will be circles surrounding the place, which therefore is the analogue of a free magnetic pole; an electron is not readily represented, nor therefore are electric currents which involve the transfer of charges through the medium. But we can have currents within a limited range; if the region contains perfect conductors, slip is continually taking place at their surfaces when disturbances are going on in the medium; this represents a current sheet around the surface of each conductor, which compensates the external disturbance and so avoids any penetration to the inside of the conductors. The model in fact illustrates the phenomena of dielectric propagation, which is necessarily of alternating character; but it does not include the free mobility of electric charges which is involved in an ordinary current of conduction. By employing wheels with various orientations and replacing the elastic bands by hydraulic connections, the model is capable of extension to the three dimensions of space.

The converse of this plan, which took the kinetic (*i. e.*, magnetic) energy of the medium to be rotatory, gives the rotationally elastic medium of Rankine and Kelvin, with kinetic energy translatory, which can be practically illustrated only by complicated gyrostatic systems. But here electrons can be definitely created in the medium by supernatural operations involving a kind of knotting process, while free magnetic poles cannot exist; and a true electric current

being a flux of electrons, the representation is complete. Not, however entirely complete; for the nature of the mobility through the medium of the core of the electron is unspecified, though when that is postulated the model shows how its surrounding electric field necessarily travels along with the core. The nature of the core itself involves the problem of the intimate intrinsic structure of the material atom: if that were explored there would be no secrets left in nature: in electro-dynamics and optics we may, therefore, be content to know the way in which an electric field travels through the æther without ever becoming disconnected from the unknown nucleus or electron to which it belongs. The science of physics is concerned only with the interactions between the atoms, which operate through the agency of the æther in which they definitely subsist; we may attain to clear views on that subject even though we be warned off the more obscure, perhaps unfathomable, problems of the intimate nature of matter itself and of the vital manifestations of which it can become the seat.

Examination of this very interesting paper will help to elucidate the fundamental part which intuition, working by means of models and analogies, can play in the formation of physical theory. According to Helmholtz this method of procedure has been characteristic of the British School. Its advantage is that correlations between directed physical quantities, which for their mathematical expression may require intricate formal operations, are made to run parallel with actual processes with which we are conversant from experience, so that they can be considerably developed without much trouble; the transition from analogy to theory then proceeds through logical analysis of these familiar processes, in the course of which ideas become generalized and in the end but little of the original analogy may remain. Notable examples are Cavendish's modes of thought which carried him some way into the nature of electrical phenomena more than a century ago, and the ideas of Faraday which formed the foundation of his own discoveries as well as the basis of the modern point of view. An intermediate link, between bare physical intuition and fully correlated theory, has been forged in the entirely new range of dynamical thought involved in Helmholtz's cardinal discovery that permanent vortex aggregates maintain their

existence (subject to the condition of stability) in a frictionless fluid medium, solely through the interaction of inertia and continuity of movement, that they in fact determine and in a sense control the entire motion of the medium to which they belong.[1] And, taking its rise from special problems of gyrostats and vortices and other steady motions, we now possess the simple and masterful generalization of the Lagrangian dynamics, foreshadowed by Kelvin and Maxwell, developed independently by Kelvin and Routh, and subsequently expounded by Helmholtz, in which the conserved momenta of permanent internal motions are considered to be a part of the constitution of the system just as much as the distribution of the masses and the geometrical configuration. So that while Lagrange and Hamilton were able to form a synthesis of the possible modes of motion of a purely dynamical system, specified as regards mass and configuration, by means of the single formula of Least Action, we are now able, as the result of a concentrated analysis, to express the laws of a dynamical phenomena that can arise in a general system more closely specified by mass and configuration and intrinsic momenta, and to lay down the limitations to which all such non-dissipative dynamical systems must conform. The question suggests itself whether, in face of so wide a synthesis, the *rôle* of analogies and models in the discovery of physical principles is not now ended: Helmholtz has in fact himself recorded that, in general physics, he found it easier to begin with wide formal relations into which the phenomena are to be fitted rather than to ascend through special analogies to more general views. Doubtless there are advances yet to come, which will arrange in still clearer and simpler order the general features of the phenomena that can arise in purely dynamical systems, and the essential limitations to which such systems are subject.

In 1882 in a paper on the electro-magnetic effects due to the motion of the Earth, FitzGerald examines the question how far Maxwell's scheme satisfies the very crucial test arising from the fact that we have here a velocity of 20 miles per second available for our

[1] For some years past FitzGerald was considerably occupied with difficult but very suggestive speculations, in part published, explaining the elasticity of the æther by a tangled structure of vortex filaments in a medium otherwise fluid.

observations.[1] In his treatise Maxwell had noticed, somewhat vaguely, that the general form of his electro-dynamic equations is not affected when the whole system is moving with uniform translatory velocity, if the axes of reference move along with the system and the optical consequences of this fact were doubtless present to his mind. If the æther were supposed to move along with the system this result would indeed be involved in the mere principles of relative motion. It is not true in any case for movements of rotation. In the present paper it is recognized that there would be an alteration in the force (of the second order however) acting between electric charges arising from their convection; for if they are convected alongside each other with the velocity of light their mutual attraction would disappear. But he now found from Maxwell's equations that there would be no effect produced on a moving magnet by any charges carried along with it, for there would be induced on its surface a charge which would exactly neutralize the magnetic influence of the moving external charges; and conversely there would be no force on the moving charges arising from currents or magnets which are carried along with them. In ignorance of this result, quite recently it has still been thought to be a difficulty in æther-theory, that no magnetic action of charges convected with the Earth's motion has been observed. In fact FitzGerald went about as far as possible at that early date: the complete analysis of convection involves separate treatment of the moving electricity and the stagnant æther, such as was developed first by Lorentz and then by Helmholtz and other writers ten years afterwards. Later on he was greatly interested in the discrepancy between Michelson's classical interference experiment, which suggested that the æther moved rigorously along with the earth, and the body of electric doctrine which almost demanded that it should be stagnant. He was the first to suggest the natural reconciliation, for which much theoretical evidence has since accrued—which is in fact a necessary consequence, if forces of cohesion are wholly or mainly electric—that motion through the

[1] Later he remarked the bearing of this velocity on any hypothesis that would make inertia a function of temperature; any such addition to mass would demand a large impulse in order to acquire for it the velocity of the Earth's motion.

æther affects the dimensions of solid molecular aggregations, but to an extent so minute as to be detectable only by most refined optical measurement.

There is a paper of this date on comets' tails. It is pointed out that Maxwell's pressure of radiation on cosmical dust, being proportional to the intensity of the radiation, is inversely as the square of the distance and so proportional to all distances to the gravitational attraction of the radiant body. For attraction thus to be changed into repulsion the mass must be very light, or its effective surface very great compared with its volume; the latter condition may be attained by external smallness of the particles. The more minute analysis that is now possible connects the repulsion directly with the reflection and absorption of the waves of radiant energy; in these processes a definite mechanical force is established whose reaction is on the surrounding æther and not on the radiant body. The validity of the principle of Least Action for the complex system æther *plus* matter implies that action and reaction are balanced for the two constituents taken together; but the pressure of radiation, now satisfactorily established on circumstantial and other evidence, shows, as Poincaré has recently admitted, that this balance cannot always be maintained for either constituent separately.

In recent years FitzGerald has taken a leading part in the critical discussions regarding the foundation of the laws of osmotic phenomena and electrolysis, and the ionization of solutions. Indeed this latter term, now universally adopted to distinguish the phenomena from ordinary dissociation, was originally due to his suggestion; last autumn, in a contribution to a British Association discussion, since abstracted *Nature*, he proposed the term electronization for the still different phenomena of molecular decomposition that are involved in the formation of the cathode streams in vacuum tubes. Some years ago he completed the work of Maxwell and Chrystal on the degree of validity of Ohm's law of conduction, by a minute test in electrolytes with powerful currents conducted through a small hole in a diaphragm; no deviation was detected, and the law possibly holds rigorously up to the point of rupture of the conducting medium.

The displacement of the lines in a spectrum towards the red,

with increasing pressure of the surrounding gas, which was discovered and investigated by the Baltimore spectroscopists, he ascribed to increase of density of the surrounding matter rather than to pressure. Each molecule in the neighborhood contributes to reduce the effective elasticity of the vibrating æthereal system, in a way which when averaged up on a large scale shows itself as increase of the electric permittivity of the medium; thus the presence of the molecule must increase all the periods of free vibration: in fact Wilsing and others have observed that when the spark is taken in dense liquid media the displacement becomes large and indefinite. He has also paid much attention to the Zeeman change produced by a magnetic field, and was the first to develop in a definite quantitative manner its relation to the Faraday magneto-optic effect.

This rapid review of the acquisition and settlement of a new region in physical science has been suggested by the recent departure from amongst us of one who by general acclaim of his fellows held the position of a leader in the enterprise. It has permitted us in a way to bring to a focus our estimate of FitzGerald's powers in the domain of physical thought. Looking back from our present position of vantage on his own published work,[1] which represented but a small portion of his scientific activity, the main feature is its tentative and pioneering character and the continuous effort towards general views; we can see more in it than his contemporaries saw at the time, doubtless more even than he was clearly conscious of himself. Although an accomplished mathematician, he preferred to reason in terms of direct images of the phenomena, and to reserve algebraic representations for purposes of calculation; he had early acquired the use of the quaternion analysis, and handled it with facility, and it was interesting to watch him getting to the bottom of a tangled scheme of physical relations by jotting down a few brief vector equations. The combined efforts of many men of genius are required to take full possession of a new territory; the subject must be for a time in the air; its ideas must be refined and polished by workers with various gifts, through the mutual attrition of apparently conflicting views, before the body of doctrine emerges

[1] It is expected that his scientific writings will shortly be republished in collected form.

in a compact and intuitive form ready to take its place as common knowledge in the scientific inheritance of mankind. Here and there giants stand out, a Maxwell or a Hertz, and by some special achievement fix the attention of their contemporaries, or sometimes it may be only of their successors. But the greater number of workers are soon merged in their generation; they have helped to mould the ideas which will be the starting-point of the next, and their place may be not the less important though no definite priorities are claimed for them, or specific discoveries attached to their names.

FitzGerald has died young, in the fiftieth year of his age, before any failure or loss of freshness had come over his mental powers. Important as have been his direct contributious to the intellectual progress of his generation, his influence over his fellows has been greater still. He was the ideal gentle knight, always more ready to afford what assistance he could to others than to proceed with his own speculations; even in his lifetime he has had his reward in becoming the trusted advisor and guide of some of the foremost scientific men of the age. His brilliant discourse to the Chemical Society of London in 1896 as the Helmholtz memorial lecture is remembered there as a classic. The impression produced by his loss may be judged from the affectionate appreciations collected recently in *Nature* (March 7) from Heaviside, Lodge, Ramsay, Perry and others of his scientific contemporaries. Here it will suffice to insert some reminiscences received more recently from Lord Kelvin.

"My first meeting with FitzGerald was when we were both guests of Dr. Traill of Trinity College, Dublin, at his country house near Bushmills, Co. Antrim, in the Easter vacation of 1883 or 1884 so far as I can remember, to visit the first electric railway of the British Empire, then recently opened between Portrush and the Giants' Causeway.

"Expecting to meet a mathematician of high order, a young and distinguished Fellow of Trinity College, I found much more than the great pleasure I anticipated. Mathematics and physics in abundance and of highly interesting quality we had, but in FitzGerald's personality there was a unique combination of which I never had experience before. His enthusiasm about water-power and electricity were delightful. We had many little and some con-

siderable troubles in reconciling steam and electric power; and whenever a linesman or a dynamo-driver (a mechanic of species then new) or a fireman or an oiler was in trouble, FitzGerald was there helping him as a comrade and acting practically as if to the manner born and educated. Our happy three days together on that occasion gave me an epitome of FitzGerald, living in an atmosphere of the highest scientific and intellectual quality, but always a comrade with every fellow-worker of however humble quality; just as I have known him ever since, and as he has been known for a quarter of a century to hosts of friends and brother workers in Ireland, England and Scotland.

"My scientific sympathy and alliance with him have greatly ripened during the last six or seven years over the undulatory theory of light and the ether theory of electricity and magnetism; and I feel keenly the scientific bereavement which his loss entailed on myself.

"As for his whole life, it seems to me that no one ever attained more nearly than FitzGerald to the chief end of man as defined by the shorter catechism of the Church of Scotland, 'to glorify God and to enjoy Him for ever.'"

NEW BOOKS.

Handbuch der Spectroscopie. Von H. KAYSER. Erster Band, Hirzel, Leipzig, 1900. Pp. xxiv + 781.

This is the first volume of one of those great treatises for the preparation of which the Germans only in our day seem to have the necessary patience and industry. To gather together all that is worth recording in the subject of spectroscopy and properly arrange and classify it is in itself a formidable task. To recognize in the confused mass of the literature those things which are really of supreme importance and to give them due prominence, subordinating without ignoring the lesser communications and treating the whole in a fair yet critical spirit, is an even more difficult one. To this work Professor Kayser has given the greater part of his time for the last ten years. The thoroughness with which the matter has been taken up may be gathered from his statement in the preface. More than forty journals and sets of transactions have been looked through from the year 1860 up to the present time and all the papers dealing with spectroscopy not merely noted but read. All references to other spectroscopic work found in these papers have been looked up and the enormous collection of material thus made has been checked by comparison with the abstracts contained in the *Fortschritte der Physik*, in *Wiedemann's Beiblätter* and in the reports of the *British Association*. More than 7,000 papers have thus been brought into review and their contents rendered readily accessible to the reader by means of a carefully prepared index of authors and an entirely separate index of subjects.

In how broad a manner the author proposes in this work to deal with his subject may be inferred from the statement of the arrangement of his treatise. Five volumes are promised us. The first, which is already at hand contains the history of spectroscopy and the description and theory of apparatus; the second volume is to deal with Kirchhoff's Law and its consequences, the emission of solids, views concerning the incandescence of gases and vapors, the dependence of emission spectra upon pressure, temperature and magnetization and the classification of spectra; the third volume is to contain an account of the phenomena of absorption together with fluorescence and phosphorescence, surface color and other allied topics; in the fourth volume each element is to be taken up separately and all that is known concerning it from the spectroscopic point of view is to be gathered together; the fifth and last volume of this exhaustive manual is intended to cover the spectroscopy of the sun and stars.

The opening chapter of Vol. I. deals as already has been indicated with the history of spectroscopy. No theme perhaps were better adapted to test the qualities of a scientific writer than this. Kirchhoff and Bunsen had numerous forerunners to some of whom the great principle which they in their work definitely and finally established was more or less clearly discernible. Concerning the matter of priority in the statement of the fundamental principles of the science a great deal has been written and nearly always in a partisan spirit. To disentangle the confused meshes of this controversy and to state fairly the services to science of the various observers who prior to 1860 attacked the great problem of the relation of the bright lines of metallic spectra to the black lines discovered by Fraunhofer in the spectrum of the sun, is indeed a difficult task. Professor Kayser has performed it in a truly admirable way. No one hitherto has succeeded in presenting the facts of the case so completely or so impartially. No one has stated so clearly the claims of tho early workers in this field nor discussed more appreciatively the bearings of their discoveries upon the development of the subject. No one perhaps in considering these early contributions, many of which led up to the very threshold of the structure built by Kirchhoff and Bunsen, has been so soundly critical. No one has brought out so clearly the relations of the earlier work to that of the two investigators at Heidelberg nor has succeeded in establishing in so convincing a manner their claim as the real founders of modern spectroscopy.

The second chapter deals with the methods of producing incandescent vapors. The author considers in succession three methods: (1) The introduction of the substances to be volatilized in various flames. (2) The use of the electric arc. (3) The electric spark discharge between electrodes of the substances to be investigated on the one hand and the electric discharge through gases and vapors at small pressures on the other. Under these heads we find a very complete discussion of the properties of flame and arc and of the phenomena of the electric discharge likewise a summary of the various practical devices employed in carrying out the three methods of obtaining gas spectra.

The exhaustive manner in which these secondary question are taken up in Professor Kayser's book finds illustration in the treatment of the question of the temperature of the Bunsen flame. The sections devoted to this subject contain a detailed account of all the important attempts to determine this temperature beginning with the original computations of Bunsen and touching upon the numerous criticisms of his results offered by contemporary authors. Crova's attempt to deduce the temperature of various sources by means of the ratio of the emission of two selected wavelengths is described and finally we have a good summary of the various measurements by means of thermo-elements, beginning with Becquerel's

work and concluding with the consistent determinations of Waggener und Berkenbusch. The various attempts by Becquerel, Rossetti, Crova, Abney and Festing, Lucas, Le Chatelier, Violle, and Wilson and Gray to determine the temperature of the electric arc and the measurements of the temperature of vapors in vacuum tubes under the electric discharge by De la Rive, E. Wiedemann, Naccari and Gugliemo, Warburg, Wood and Graham are discussed in an equally satisfactory manner. The numerous forms of vacuum tube applied in spectroscopy are described at considerable length and this is followed by an admirable summary of the methods employed by various investigators to secure gas spectra free from the disturbing influence of the vapors of mercury, of the hydrocarbons, of water, etc. Kayser points out that in general the vapor of mercury introduces no confusion on account of the well-established character of its spectrum and that the same is true of water vapor which gives only the well-established lines of hydrogen. Mercury and hydrogen lines indeed afford a certain advantage in that they give convenient reference points in the spectrum.

The chapter on prisms was written at the author's request by Dr. Konen. It constitutes, taken by itself, a very complete treatise on this subject and is rendered especially useful by means of a bibliography in which some 263 titles are given. The discussion is not confined to the action prisms in the visible spectrum but is extended so as to include the ultra-violet and the infra-red. The performance of rock salt, fluorite, sylvin and quartz both as regards transmitting power and dispersion are considered and the constants for these materials from the works of Schumann, Sarasin, Rubens, Paschen and others are given in tabular form. This chapter, which occupies 140 pages, is followed by an almost equally exhaustive one on the diffraction grating.

The fifth chapter of Vol. I. is entitled *Spectroscopic Apparatus.* It opens with the description of instruments for the study of the visible spectrum, beginning with the classical spectroscopes of Kirchhoff and Bunsen, and leads up in historical order to the apparatus based upon the interference methods of Michelson. The limitations of visual work are fully considered in a section devoted to the sensitiveness of the eye and this is followed by a description of the application of photography to the study of the spectrum. The treatment of photographic plates for the purposes of spectrum photography in the ultra-violet and in the infra-red is given in sufficient detail to make this portion of the volume a practical handbook. The methods by which Abney on the one hand succeeded in the photography of the infra-red and Schumann on the other extended our knowledge of the ultra-violet to wave-lengths hitherto unknown are given due prominence. The chapter closes with an account of the various methods for the study of the spectrum by means

of its heating effect. The thermopile, the bolometer, the microradiometer by Boys and the radiometer of Ernest Nichols are briefly discussed and their relative merits compared. The concluding chapter of volume I. deals with the attempts to establish the wave-lengths of the chief reference lines in the spectrum in absolute measure together with the important matter of the calibration of prisms and the reduction of measurements made upon prismatic spectra to the normal scale.

If Professor Kayser succeeds in carrying his great plan to its completion, science will be indebted to him to an extraordinary degree. All those who are interested in spectroscopy will look forward eagerly to the appearance of the subsequent volumes of his great treatise.

E. L. N.

The Elements of Hydrostatics. By S. L. Loney. Pp. 248 + xxii. Cambridge, 1900. New York, The Macmillan Co.

This little book forms an elementary treatise on the subject of hydrostatics, with special reference to its practical applications. It will recommend itself especially to students who possess a limited knowledge of mathematics, and who would find the works of Greenhill or Minchin beyond their reach. Nothing further than elementary algebra, geometry and trigonometry, and the elements of statics is presumed on the part of the reader.

The first few chapters are devoted to the discussion of the fundamental magnitudes, pressure and density. Then follow in succession chapters on the equilibrium of floating bodies, on methods of determining specific gravity, on gases, and on machines and instruments which illustrate the properties of fluids. In this chapter the hydrostatic press, the diving bell, pumps, etc., are treated. In the remaining chapters of the book such subjects are discussed as centers of pressure, rotating liquids, curves of buoyancy and tension of containing vessels.

An excellent feature of the book is its careful illustration of important principles; the demonstrations of the theorems are nearly always accompanied by illustrative examples, and followed by exercises. Numerous exercises, most of which are easy, are placed also at the ends of the chapters; the harder exercises are conveniently indicated. Most of the examples are numerical; general constants, which are apt to convey no definite meaning to beginners, are used only sparingly.

It is surprising that the subject of capillarity should be entirely neglected; it is mentioned but once in the whole book, in a single short sentence on barometer corrections. While an extended treatment of capillary phenomena would be outside of the purpose of the book, some mention might have been made of the behavior of liquid surfaces at the walls of

the containing vessels. As it is, the reader, if not already acquainted with the subject, would certainly obtain the impression from the unmodified statements of the book and the figures, that the surface of a liquid under the action of gravity is horizontal throughout its entire extent.

In general it may be said that the book deserves recommendation as a clear and concise treatment of the essential principles of hydrostatics without the use of higher mathematics.

JOHN R. BENTON.

Signalling Through Space Without Wires. By OLIVER J. LODGE. Pp. 133.

There is before us an enlarged edition of "The Work of Hertz and his Successors" by Oliver J. Lodge. The first edition has already been reviewed in this journal. The present one is published under the title "Signalling Through Space Without Wires," and adds to the former edition a most useful résumé of the work done during the past few years on "wireless telegraphy." There are 72 pages in the added matter making a book of 133 pages. The new matter consists of a history of the coherer principle, the applications which Marconi and others have made of this, and of Professor Lodge's own work on syntonic telegraphy. It gives a most interesting and judicious statement of our present knowledge of this subject and well deserves a wide and thorough reading.

C. D. CHILD.

School Chemistry. By JOHN WADDELL. New York, The Macmillan Co., 1900. Pp. 278.

This book is evidently designed for a laboratory manual, supplementing the class-room work of the teacher. Starting with the study of the most common substances, the student discovers their properties, reactions and composition, and from the facts accumulated is able to understand the clear enunciation and explanation of the laws of chemistry. He then studies in detail the most important elements and compounds.

The long and varied teaching experience of the author has impressed on him the necessity of questioning the student repeatedly during the experiment and of showing clearly the relation between facts, laws and theories; his methods illustrate this.

The book does not presuppose a sufficient knowledge of physics or physical methods on the part of the student, nor are the many interrelations of the two branches considered. Until this is done, quantitative experiments in such a work are useless.

The cuts in the book are not good; otherwise it is neatly and conveniently arranged. It will certainly be well received by students and teachers of chemistry.

H. R. CARVETH.

A Treatise on Qualitative Chemical Analysis. By J. F. SELLERS. Boston, Ginn & Co., 1900. Pp. 157.

The author, realizing that the number of books on qualitative analysis is already legion, makes an extensive apology in the preface for his own. Almost every teacher can appreciate the fact that one's own method of presentation seems the best. Some useful hints are given in the chapters upon the theory and methods of analytical operations and separations. The discussion of the effect of ammonium salts and the treatment of the barium group is especially noteworthy. We should not hesitate to assign this book a place among the better class of works upon qualitative analysis.

PHILIP E. BROWNING.

A Treatise on Elementary Dynamics, dealing with relative motion mainly in two dimensions. By H. A. ROBERTS, M.A. Pp. 258 + xii. London, Macmillan & Co.

In the preface of this book the author expresses the hope "that this little treatise may be of use to candidates for mathematical scholarships, and to others whose reading is not quite elementary." The work is designed primarily for students in England, where it is customary to take a thorough course in elementary mechanics before entering upon the study of higher mathematics. A knowledge of the calculus is, therefore, not assumed on the part of the reader. In this country, where the arrangement of the curriculum is somewhat different, the book would not be so well adapted as some others to the needs of most students beginning mechanics. That it will be of use to the "others whose reading is not quite elementary," however, there can be no doubt, for it presents the subject-matter in a manner at once very scientific and very different from that of the books most used in this country.

In this book vector methods are employed almost exclusively, and the Cartesian treatment of the subject is made decidedly subordinate; while the reverse is the case in the books in most general use. The treatment is throughout clear and rigorous, and the arrangement logical.

It is possible that the contents of the book would have been better described if it had been entitled "Dynamics of a Particle," for although the dynamics of rigid bodies is not altogether neglected, less prominence is given to it than might be expected in a work on general dynamics. The rotation of rigid bodies, for example, receives no systematic treatment. A discussion of couples and their representation by means of vectors would have been very appropriate to the methods of the book; yet the author goes no further than to give the definition of a couple.

It may be said that the book is a valuable addition to the working literature of the subject, as it forms an excellent introduction to the

study of vector analysis, and furnishes many elegant examples of applications of that method.

J. R. BENTON.

Album di Elettricisti Italiani ed Esteri. Milan, 1900.

A new edition of the Album of Italian and Foreign Electricians has just been issued by the Italian journal *Elettricita.* It contains 240 half-tone reproductions, chiefly from photographs, although the portraits of some of the older electricians, such as Volta, Galvani and Faraday, are from wood cuts or engravings.

This collection affords an illustration of the difficulty of securing uniformly satisfactory results by copying in half-tone a miscellaneous lot of ordinary photographic prints. There are, however, many excellent likenesses and a few strikingly good portraits.

E. L. N.

Introduction to Science. (The Temple Primers.) By ALEX. HILL. 16mo. Pp. 140. New York, The Macmillan Co., 1900.

This little volume contains the following chapters:

Definition of Science; Aim of Science; Boundaries of Science; The Relation of Philosophy to Science; The Senses the Agents of the Mind; The Extension of the Senses by Artificial Aids; Classification of the Sciences; History of Science; Method of Science; The Age of the Earth; The Ultimate Constitution of Matter; The Origin of Species; The Cause of the Coagulation of Blood; The Function of Nerve-fibers and Nerve-cells; Microphytology.

Observations Pluviometriques et Thermometriques Faites Dans Le Departement De La Gironde de Juin 1898 a Mai 1899. Note De M. G. RAYET. Pp. 73. Bordeaux, Imprimerie G. Gounouilhou. 1899. (*Received.*)

Philosophy of Voice. By CHARLES LUNN. Pp. 189. Bailliere, Tindall, and Cox, London, 1900. (*Received.*)

U. S. Department of Agriculture. Report 64. Field Operations of the Division of Soils, 1899. By MILTON WHITNEY. Pp. 198. Washington, Gov. Printing Office. 1900. (*Received.*)

Alternating Currents. An Analytical and Graphical Treatment for Students and Engineers. By FREDERICK BEDELL and ALBERT C. CREHORE, Fourth Edition, 8vo, pp. 325. Cornell Coöperative Society, Ithaca, N. Y., 1901. (*Received.*)

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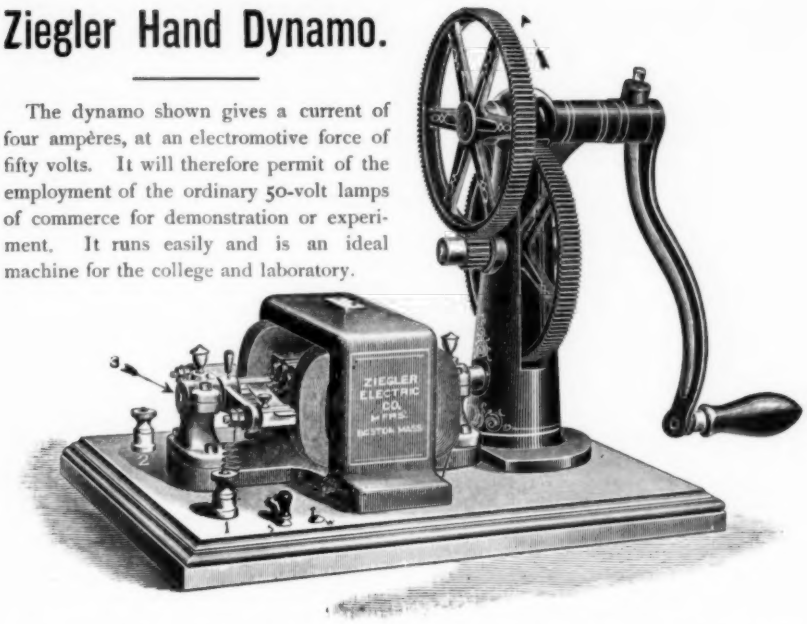

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Contents of Recent Numbers

JULY 1900.

On the Spectrum of Radium C. RUNGE

The Velocity of Meteors as Deduced from Photographs at the Yale Observatory W. L. ELKIN

Some New Fluorescence and Afterglow Phenomena in Vacuum Tubes Containing Nitrogen . . . . . PERCIVAL LEWIS

The Effect of Some Impurities on the Spectra of Some Gases. II. PERCIVAL LEWIS

Prominences Observed at Kalocsa on May 28, 1900 . . . . . J. FÉNYI

A Suggested Explanation of the Solar Corona . . . . . J. SCHEINER

Remarks on the Construction and Adjustment of Spectrographs. II. J. HARTMANN

Note on the Spectrum of Silicon FRANZ EXNER and EDUARD HASCHEK

Pressure in the Spark EDUARD HASCHEK and HEINRICH MACHE

James Edward Keeler GEORGE E. HALE

Minor Contributions and Notes: A New Star in *Aquila*, E. C. PICKERING; The Yerkes Observatory of the University of Chicago, *Bulletin* No. 13, GEORGE E. HALE; Positions of *Eros* (433) in 1893, 1894, and 1896 EDWARD C. PICKERING

Eclipse Reports: Preliminary Results of the United States Naval Observatory Eclipse Expeditions, S. J. BROWN; A Preliminary Statement of the Results of the Smithsonian Observatory Eclipse Expedition, C. G. ABBOT; Eclipse Observations by the Princeton Party at Wadesboro, N. C., May 28, 1900, C. A. YOUNG; The Yerkes Observatory of the University of Chicago, *Bulletin* No. 14, Observations of the Total Solar Eclipse of May 28, 1900, at Wadesboro, N. C., GEORGE E. HALE; Observations of the Total Eclipse at Centerville, Norfolk County, Va., WINSLOW UPTON; Observations at Juliette, Ga., W. CROCKETT; Observations of the Total Solar Eclipse of May 28, 1900, by the Harvard Observatory Expedition, WILLIAM H. PICKERING; Observations of the Total Eclipse by the Vassar College Party, MARY W. WHITNEY; Observations by the Massachusetts Institute of Technology Party, ALFRED E. BURTON; Observations at Wadesboro, N. C., J. B. COIT; Observations at Southern Pines, N. C., F. E. SEAGRAVE; The Weather Bureau Observations of the Total Eclipse of May 28, 1900; Observations at Norfolk, Va., FRANK W. VERY; Observation by the Georgetown College Party, J. G. HAGEN; Persistence of the Corona After Totality, JOHN FRED MOHLER.

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The Spectra of Mercury WILLIAM B. HUFF

An Investigation on the Zeeman Effect HERBERT M. REESE

A New Theory of the *Milky Way* C. EASTON

Minor Contributions and Notes: Variable Stars in Clusters. Rate of Increase of Light, EDWARD C. PICKERING; The Yerkes Observatory of the University of Chicago, *Bulletin* No. 15, GEORGE E. HALE; Comparison of a Prism and a Grating Spectroscope, N. ERNEST DORSEY.

OCTOBER 1900.

On the Arc-Spectra of Some Metals as Influenced by an Atmosphere of Hydrogen HENRY CREW

Some Abnormal Stars in the Cluster *M 13 Herculis* . . . . . E. E. BARNARD

Discovery and Period of a Small Variable Star in the Cluster *M 13 Herculis* E. E. BARNARD

Solar Phenomena, Considered in Connection with Anomalous Dispersion of Light W. H. JULIUS

Note on Inquiries as to the Escape of Gases from Atmospheres G. JOHNSTONE STONEY

The Complete Emission Function P. G. NUTTING

Rise of a Large Prominence on June 1, 1900 J. FÉNYI

Field of the Reflecting Telescope S. C. REESE

Reviews: A general catalogue of 1290 double stars discovered from 1871 to 1899 by S. W. Burnham, S. W. BURNHAM (E. E. Barnard); Photometric Revision of the Harvard Photometry during the years 1891–1894, EDWARD C. PICKERING (J. A. P.)

JANUARY 1901.

On the Period of the Solar Spots SIMON NEWCOMB

On an Apparatus for the Laboratory Demonstration of the Doppler-Fizeau Principle . . . . A. BÉLOPOLSKY

The Radiation of a Black Body C. E. MENDENHALL and F. A. SAUNDERS

Examination of *Pleiades* and *Eros* Plates Taken with the Crossley Reflector of the Lick Observatory

On the Production of a Line Spectrum by Anomalous Dispersion, and its Application to the "Flash Spectrum" R. W. WOOD

The Nature of the Solar Corona R. W. WOOD

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LXI. March, 1901

TABLE OF CONTENTS

The Various Determinations of the E.M.F. of the Clark Cell. HENRY S. CARHART.

Velocity of Ions drawn from the Electric Arc. C. D. CHILD.

A Spectrophotometric Study of the Hydrolysis of Dilute Ferric Chloride Solutions. B. E. MOORE.

Coherers Suitable for Wireless Telegraphy. CARL KINSLEY.

A Graphical Method for Analyzing Distorted Alternating Current Waves. A. S. LANGSDORF.

New Books: *Stine:* Photometrical Measurements.

LXII. April, 1901

TABLE OF CONTENTS

Experimental Studies on the Resonance of Conical, Trunco-Conical and Cylindrical Air-columns. VICTOR MAHILLON.

On the Freezing-Points of Aqueous Solutions of Non-Electrolytes. II. E. H. LOOMIS.

The Formula for the Depression of the Freezing Temperature of Solutions. W. F. MAGIE.

On the Action of the Coherer. K. E. GUTHE.

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